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Spite and Counter-Spite in Auctions

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Spite and Counter-Spite in Auctions*

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Abstract

The paper presents a complete information model of bidding in second price sealed bid and ascending price (English) auctions, in which potential buyers know the unit valuation of other bidders and may spitefully prefer that their rivals earn a lower surplus. Bidders with spiteful preferences should overbid in equilibrium when they know their rival has a higher value than their own, and bidders with a higher value underbid to "counter" spite the overbidding of the lower value bidders. The model also predicts different bidding behavior in second price as compared to ascending price auctions. The paper also presents experimental evidence broadly consistent with the model. In the complete information environment, lower value bidders overbid more than higher value bidders, and they overbid more frequently in the second price auction than in the ascending price auction. Overall, the lower value bidder submits bids that exceed value about half the time. These patterns are not found in the incomplete information environment, consistent with the model.

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1. Introduction

One of the most basic and apparently innocuous assumptions about behavior in games is that players will adopt dominant strategies. If this assumption has only limited predictive power, however, it calls into question the empirical relevance of many important concepts in mechanism design, such as strategy-proofness. Recent laboratory research in public good mechanism design has documented extensive failure by subjects to follow dominant strategies even in fairly simple environments (Attiyeh et al., 2000; Kawagoe and Mori, 2001; Cason et al., 2006). Experiments have also shown that subjects do not bid optimally in incentive-compatible second-price (Vickrey) auctions. For example, Kagel and Levin (1993) find that 58 to 67 percent of bids exceed value, and Harstad (2000) reports that severe overbidding does not decline over time.

Overbidding is much less pronounced in the (isomorphic) English, ascending price auction. The equilibrium bidding strategy is more transparent in the ascending-price auction, which has led some researchers to conclude that the subtlety of the dominant strategy in the sealed bid second-price auction is a primary reason bidders fail to follow it. Learning is also difficult in the second-price auction because the use of a weakly dominated strategy may often not cause any loss in actual payoff (Kagel and Levin, 1993). Moreover, even with standard (own-payoff maximizing) preferences, many Nash equilibria exist in these auction formats other than the dominant strategy equilibrium. But what if the dominant strategy equilibrium does not exist because preferences are misspecified?

The paper explores an alternative explanation for overbidding in second-price and ascending-bid auctions: spiteful preferences. A spiteful agent has utility that increases when the earnings of her rivals decrease. With such preferences, a spiteful agent may be willing to sacrifice her monetary payoff in order to reduce the other agent's monetary payoff. The following section contains our formal definition, which features a reciprocal motive; i.e., subjects feel more spiteful towards others who treat them spitefully. The key design feature of second-price and ascending-price auctions that make them incentive-compatible under standard (own-payoff maximizing) preferences makes them particularly prone to manipulation by bidders who have spiteful preferences. Because an individual's monetary payoff conditional on winning the auction is independent of her bid, if she cares only about her monetary payoff has no incentive to manipulate her bid to lower her price. But if she fails to win her bid may determine the payoff of the winner. Therefore, if she is spiteful she can increase her bid to increase her (spiteful) utility.

Agents who have spiteful preferences would not consider a bid equal to value to be a dominant strategy.

We construct a two-bidder, intention-based sequential decision model which shares its sprit with Dufwenberg and Kirchsteiger (2004). Intention is measured by the distance between two monetary payoffs, one generated by a chosen bid and the other by a value-revealing bid. An auction is a market that has clear winners and losers. A bidder with lower value may behave spitefully when she infers her opponent's intention to win a positive winning surplus. This can be interpreted as part of the disutility of losing the auction, since she is in a disadvantageous position. This prompts her to place a spiteful bid higher than her value, hoping to give her opponent a negative psychological payoff in addition to reducing his winning surplus. A novelty of our analysis is in incorporating retaliation by a bidder with higher value. A spiteful bid by the lower value bidder can backfire by inducing the higher value bidder to place a deliberately low bid in order to penalize the spiteful conduct by the lower value bidder, even though such a retaliatory bid reduces his chance of winning.

In particular, we consider a complete information environment that strengthens the impact of social preferences such as spite and reciprocity. In the standard incomplete information environment typically employed in the auction literature, adding spiteful preferences as we have modeled them still results in bids equal to value in the unique symmetric equilibrium. By contrast, bidders with spiteful preferences should overbid in equilibrium when they have complete information about their rival's value and they know their rival has a higher value than their own. Bidders with the higher value underbid to "counter" spite the overbidding by the lower value bidders. Spiteful preferences also make the two auction forms non-isomorphic. In an ascending-price auction, an auctioneer or clock raises a calling price until there remains only one active bidder. A climbing calling price gradually reduces the winner's payoff. Taking this effect into account, in our sequential decision model the bidders revise their estimates upward about the other's spitefulness when they arrive at each new, higher calling price. This makes the bidder with the higher value willing to retaliate at earlier stage. Consequently, for the same level of spiteful preferences, lower value bidders should overbid less in the ascending-price auction than in the second-price auction in this environment. Thus, the set of equilibria in ascending-price auctions is smaller than in second-price auctions.

We also present experimental evidence broadly consistent with the predictions of this model. In the complete information environment, lower value bidders overbid more than higher value bidders, and they overbid more frequently in the second price auction than in the ascending price auction. Overall, the lower value bidder submits bids that exceed value about one-half the time. These patterns are not found in the data we collected for the incomplete information environment, consistent with the model.

Researchers have recently measured and explored the impact of social preferences that include reciprocity and spite in a variety of environments, but often in non-competitive contexts such as public good provision, two-agent bargaining and simple games.[1] A small amount of research has studied the impact of spite in auctions, starting with Morgan et al.'s (2003) theoretical analysis (which we became aware of after starting the research reported here). Their model, which we discuss later in more detail, features non-reciprocal spite and does not predict differences between the second price and ascending price auctions for the two-bidder setting we employ. Cooper and Fang's (2007) experimental study also considers (like us) a two-bidder environment for simplicity, but only second price auctions. They provide bidders with noisy information about their rival's value, with varying degrees of accuracy, and find that overbidding is consistent both with spite and "joy-of-winning" motivations. Andreoni et al. (2007) also report a laboratory experiment in which bidders may have information about rivals' value draws. They consider first and second price auctions, all with four competing bidders, and test predictions regarding equilibrium strategies in three different information structures. Their results provide strong support for theory, but they also observe overbidding by lower value bidders in their second price auctions that is consistent with a spite motive.

Our results are also consistent with spiteful bidder preferences, and we observe overbidding and underbidding in a pattern consistent with our model of reciprocal spite. Lower value bidders overbid relative to their values, but in response the higher value bidders underbid to punish this overbidding (or at least make overbidding risky). Spite leads to counter-spite, and in equilibrium these spiteful social preferences substantially reduce the size of the set of Nash equilibria. Moreover, this combination of spite and counter-spite is the reason that isomorphism fails for the second price and ascending price auction, and the particular pattern of larger and more frequent overbids in the second price auction predicted by the model is also observed in the experimental data.

[1] Many preference models now exist that extend the utility domain beyond own monetary payoff to include psychological payoffs, often to explain unconventional other-regarding behavior in non-market contexts. Those include Rabin (1993) based on a psychological game proposed by Geanakoplos, Pearce, and Stacchetti (1989), Levine (1998), Fehr and Schmidt (1999), and Falk and Fischbacher (2006).

2. Theory

Although our assumption of complete information about rivals' values in the following two subsections is admittedly extreme, it is consistent in spirit with the increased information about rivals studied in the recent contributions of Andreoni et al. (2007) and Cooper and Fang (2007) that also consider implications of spite in auctions. In practice, bidders in many situations would have some, perhaps noisy, information about rivals' trading interests. Our complete information model provides a useful benchmark for the case of fully-informed bidders, which presents the starkest contrast to the more standard incomplete information context, considered below in Section 2.3. In this section, we present the details of our models and characterize equilibrium bidding behavior. All proofs are collected in Appendix A.

2.1 The Model with Known Values (Complete Information)

There is one seller having one unit of good to sell, whose benefit from retaining it is zero. We consider two risk neutral buyers participating in an ascending-bid auction. Each buyer $i \in \{1,2\}$ has unit demand and values that unit privately. The value measurement is in terms of transaction unit $\varepsilon > 0$, corresponding for example to a minimum currency unit. Thus, each individual value is denoted by $v_i \in V$, where $V = \{0, \varepsilon, 2\varepsilon, \cdots, \overline{v} - \varepsilon, \overline{v}\}$, $\overline{v} = \overline{u}\varepsilon$ and $\overline{u} \in N \setminus \{1\}$, where N is a set of natural numbers. We assume that each buyer knows each other's value and that $v_1 > v_2$ without loss of generality.

In an ascending-bid auction, an auctioneer (or auction clock) cries out a calling price. Once the auction starts, the calling price gradually rises by unit ε. All buyers are assumed to be active at the start, and the auction terminates as soon as only one active buyer remains. When both buyers withdraw simultaneously, the winner is chosen randomly with equal probabilities, and the winner has to pay her own withdrawal bid. Let $B = \{0, \varepsilon, 2\varepsilon, \cdots, \overline{b} - \varepsilon, \overline{b}\}$ be a set of withdrawal bids commonly available to the two buyers $B = B_1 = B_2$, where $\overline{b} = \lambda\varepsilon > \overline{v}$ and $\lambda \in N \setminus \{1\}$. Let $r \in B$ denote the calling price, and $r = 0$ corresponds to the initial stage before the auction starts. Clearly the decision problem each buyer faces at the start of the auction, when $r = 0$, exactly corresponds to the second price sealed-bid auction. At every calling price level r including 0, buyers make simultaneous decisions as to whether to drop out immediately from the bidding, or stay in and make a future withdrawal plan. As the calling price rises by ε, buyers move to a new decision point with $r+\varepsilon$, where they are no longer allowed to make or plan a withdrawal bid in the range $\{0,\cdots,r-\varepsilon,r\}$. Let us denote the withdrawal bid made in a given decision point with a calling price at *r* as $b_i^r \in B_i^r = \{r, r+\varepsilon,\cdots,\bar{b}\}$, $i \in \{1,2\}$.

At each decision point, buyer *i* estimates *j*'s withdrawal bid $b_j^r \in B_j^r$, and uses it as her expected payment assessment at *r*. The final decision point where one of the bidders actually withdraws can be identified as *r* satisfying $\min\{b_1^r, b_2^r\} = r$. We refer to those decision nodes leading to the final decision node as interim decision points. A bidding strategy is a sequence of actions $a_k \in \{0,1\}$, $k \in \{0,\varepsilon,\cdots,r,\cdots,\bar{b}\}$, where $a_k = 1$ represents an action to stay active in the auction and $a_k = 0$ is an action to drop out of the auction. A bidder *i*'s strategy made at *r* to withdraw at b_i^r is a sequence of $a_k = 1$, for all $k \le b_i^r$, and $a_k = 0$ for all $k > b_i^r$. The set of feasible strategies $S_i \subset \Re^{\bar{b}+1}$ for buyer *i* consists of only sequences of the form $\{1,1,\cdots,1,0,0\cdots0\}$ such that $a_k = 1$ for all $k \le n$ and $a_k = 0$ for all $k > n$, for all $n \in \{\varepsilon,\cdots,r,\cdots,\bar{b}\}$. All strategies lead buyers to stay in the auction until the calling price hits *n* and withdraw there once for all, because the rule of the ascending-bid auction does not allow a buyer to reenter the auction once she withdraws. Consequently $S_1 = S_2$. Let a function $s_i : B_i^r \to S_i$, $i \in \{1,2\}$, $i \ne j$, denote a bidding strategy and $s_i(b_i^r)$ represents bidder *i*'s strategy to withdraw at b_i^r.

Departing from the conventional view of economic agents, we consider buyers who receive some psychological payoff in addition to a monetary payoff. The psychological payoff includes utility both from spite bidding and from retaliating against spiteful bids. Each buyer estimates her opponent's spiteful intention by her opponent's choice of withdrawal bid. To analyze such intention-based decision making, we will construct a sequential decision model in a spirit of Dufwenberg and Kirchsteiger (2004), utilizing two layers of beliefs, with modifications to apply the context of the ascending-bid auction.[2] For $i, j \in \{1,2\}, i \ne j$, and in a given decision point *r*, let $b_{ij}^r \in B_j^r$ denote buyer *i*'s ex ante assessment as to when buyer *j* would withdraw from

[2] Our study shares the spirit with other types of model with reciprocity, such as those proposed by Rabin (1993) and Falk and Fischbacher (2006).

bidding, and let and $b_{iji}^{r} \in B_{i}^{r}$ denote buyer i's assessment of when buyer j expects buyer i to withdraw. Let $s_{ij}(b_{ij}^{r}) \in S_{j}$ and $s_{iji}(b_{iji}^{r}) \in S_{i}$ denote a corresponding belief assessment of strategy of j and i, where the former is labeled as buyer i's first order belief and the latter as i's second order belief at r. Those beliefs are maintained unless they become inconsistent with rising r, that is, when either $b_{ij}^{r-\varepsilon} < r$ or $b_{iji}^{r-\varepsilon} < r$ occurs.

Under the set of first and second order beliefs, buyer i estimates her payoff, which consists of her monetary payoff and psychological payoff. Her payoff is defined on her own strategy choice and a set of beliefs $(s_{i}(b_{i}^{r}), s_{ij}(b_{ij}^{r}), s_{iji}(b_{iji}^{r}))$. These variables ultimately depend on $(b_{i}^{r}, b_{ij}^{r}, b_{iji}^{r})$, however, and in what follows, we will define buyers' monetary and psychological payoff in reduced form to make its expression simple. We also use the term "beliefs" to refer to $(s_{ij}(b_{ij}^{r}), s_{iji}(b_{iji}^{r}))$ and $(b_{ij}^{r}, b_{iji}^{r})$ interchangeably when there is no fear of confusion. Buyer i's monetary payoff at a given decision point r is given by a function $\pi_{i} : B_{i}^{r} \times B_{j}^{r} \to \Re$, $i, j \in \{1,2\}, i \neq j$, such that

$$\pi_{i}(b_{i}^{r}, b_{ij}^{r}) = (v_{i} - b_{ij}^{r}) I_{b_{i}^{r} > b_{ij}^{r}} + 0 \cdot I_{b_{i}^{r} < b_{ij}^{r}} + (1/2)(v_{i} - b_{ij}^{r}) I_{b_{i}^{r} = b_{ij}^{r}},$$

where I_{A} is an index function which assumes value 1 when the statement A holds, and zero otherwise. The first term is her expected winning monetary payoff, the second term is her losing monetary payoff of zero, and the third term is her expected payoff from a tie.

Buyers' psychological payoff has a multiplicative form to capture intention-based reciprocal interaction between buyers. To better understand this specific form, consider the case of buyer 2 at an interim decision point r when she chooses a withdrawal bid $b_{2}^{r} \in B_{2}^{r}$, under the first order and second order beliefs of $b_{21}^{r} \in B_{1}^{r}$ and $b_{212}^{r} \in B_{2}^{r}$. The ascending-bid auction rules imply that buyer 2's withdrawal bid choice $b_{2}^{r} \in B_{2}^{r}$ determines buyer 1's monetary payoff. Buyer 2's anticipation of buyer 1's payoff at an interim decision point r is $\pi_{1}(b_{21}^{r}, b_{2}^{r}) = (v_{1} - b_{2}^{r}) I_{b_{21}^{r} > b_{2}^{r}} + 0 \cdot I_{b_{21}^{r} < b_{2}^{r}} + (1/2)(v_{1} - b_{2}^{r}) I_{b_{21}^{r} = b_{2}^{r}}$. Recall that buyer 2 knows that her value is less than the value of buyer 1; consequently, she expects to lose. Buyer 2 can also expect that buyer 1 feels entitled the full winning payoff of $v_{1} - v_{2}$, based on the conventional dominant strategy equilibrium where both buyers are not spiteful and bid equal to their own values. Since buyer 1's entitled winning payoff is $v_1 - v_2$ at every decision point, a reasonable reference payoff for buyer 1 from buyer 2's point of view is $\hat{\pi}_1(b_{21}^r, b_2^r) = (v_1 - v_2)I_{b_{21}^r > b_2^r} + 0 \cdot I_{b_{21}^r < b_2^r} + (1/2)(v_1 - v_2)I_{b_{21}^r = b_2^r}$.

If buyer 2 is spiteful, she would plan to withdraw at b_2^r higher than v_2 but less than v_1 so as to let buyer 1 win with a positive but smaller winning payoff $v_1 - b_2^r$. The distance between the buyer 1's actual payoff, $\pi_1(b_{21}^r, b_2^r)$ and her reference payoff $\hat{\pi}_1(b_{21}^r, b_2^r)$ reflects the intensity of buyer 2's desire to harm buyer 1. In particular, we use this distance, relative to what payoff range buyer 1 would expect to be relevant for her psychological assessment. The spiteful withdrawal bid of buyer 2 that buyer 1 should pay particular attention to lies in the range $[\max\{r, v_2\}, v_1]$. Assuming buyer 2 is being spiteful, at each interim decision point the maximum winning payoff available for buyer 1 is $\overline{\pi}_1^r = v_1 - \max\{r, v_2\}$, and the minimum is $\underline{\pi}_1^r = v_1 - v_1 = 0$. When buyer 2 wins, buyer 1 receives the losing payoff of zero. Thus, we measure the intensity of buyer 2's spite intention toward buyer 1 at *r* by a function $f_2^r : B_2^r \times B_1^r \to \Re$ defined for $r \le v_1$ as the following ratio:

$$f_2^r(b_2^r, b_{21}^r) = \frac{\pi_1(b_{21}^r, b_2^r) - \hat{\pi}_1(b_{21}^r, b_2^r)}{\overline{\pi}_1^r - \underline{\pi}_1^r + \varepsilon}$$

$$= \frac{(v_2 - b_2^r)I_{b_{21}^r > b_2^r} + 0 \cdot I_{b_{21}^r < b_2^r} + (1/2)(v_2 - b_2^r)I_{b_{21}^r = b_2^r}}{v_1 - \max\{r, v_2\} + \varepsilon}. \quad (2.1)$$

A negative value of f_2^r means buyer 2 being spiteful toward buyer 1. Note, in particular, when the calling price *r* climbs beyond v_2, the denominator of f_2^r is revised as a new narrower payoff range $\overline{\pi}_1^r - \underline{\pi}_1^r$. This increases f_2^r in absolute value. When $r \ge v_1 + \varepsilon$, we set $f_2^r(b_2^r, b_{21}^r) = 0$, because it is hard to interpret buyer 2 bidding beyond v_1 as a product of spite intention toward buyer 1.[3]

In a similar but slightly different manner, buyer 2 assesses buyer 1's intention, which is measured by a function $\hat{f}_{21}^r : B_1^r \times B_2^r \to \Re$, such that

[3] The ε is added to the relevant spite payoff range $\overline{\pi}_1^r - \underline{\pi}_1^r$ in the denominator of (2.1) to prevent it from vanishing when $r = v_1$.

$$\hat{f}_{21}^{r}(b_{21}^{r}, b_{212}^{r}) = f_{21}^{r}(b_{21}^{r}, b_{212}^{r}) + \delta_{21}^{r}(b_{21}^{r}, b_{212}^{r}). \tag{2.2}$$

The first component is an index analogous to f_2^r in (2.1). At each decision point r, buyer 2 expects buyer 1 to withdraw at b_{21}^r, and she anticipates that buyer 1 expects buyer 2 to withdraw at b_{212}^r. The reference payoff of buyer 2 should be zero, $\hat{\pi}_2 = 0$, either when she wins or when she loses, based on the value-revealing reference bidding strategy. Expecting that buyer 1's counter spiteful withdrawal bid b_{21}^r could range from $[\max\{r, v_2\}]$ to v_1, the relevant winning payoff ranges from $\underline{\pi}_2^r = v_2 - v_1$ to $\overline{\pi}_2^r = v_2 - \max\{r, v_2\}$. As before, we measure buyer 1's intensity of spitefulness, from buyer 2's perspective, by a ratio of the distance between buyer 2's payoff $\pi_2(b_2^r, b_{21}^r)$ against the relevant payoff range, which is captured by a function $f_{21}^r : B_1^r \times B_2^r \to \Re$ as

$$\begin{aligned} f_{21}^{r}(b_{21}^{r}, b_{212}^{r}) &= \frac{\pi_2(b_{212}^{r}, b_{21}^{r}) - 0}{\overline{\pi}_2^r - \underline{\pi}_2^r + \varepsilon} \\ &= \frac{0 \cdot I_{b_{21}^r > b_{212}^r} + (v_2 - b_{21}^r) I_{b_{21}^r < b_{212}^r} + (1/2)(v_2 - b_{21}^r) I_{b_{21}^r = b_{212}^r}}{v_1 - \max\{r, v_2\} + \varepsilon}. \end{aligned} \tag{2.3}$$

The index f_{21}^r does not affect utility when buyer 2 loses and obtains her zero reference payoff. As in the case of f_2^r in (2.1), it is not reasonable to interpret any counter spite intention behind any bid exceeding v_1, in which case we set f_{21}^r=0.

If buyer 1's intentions matter to buyer 2, buyer 2 should feel differently about the same losing monetary payoff depending upon buyer 1's choice of withdrawal bid, since it reflects buyer 1's retaliatory intentions as well as her attempt to secure a winning payoff of at least $v_1 - b_{21}$. This is reflected in the second component of her psychological payoff, which is a function $\delta_{21}^r : B_1^r \times B_2^r \to \Re$, defined by

$$\begin{aligned} \delta_{21}^{r}(b_{21}^{r}, b_{212}^{r}) &= \frac{0 \cdot I_{b_{21}^r < b_{212}^r} + [-\max\{v_1 - b_{21}^r, 0\}] \cdot I_{b_{21}^r > b_{212}^r} + (1/2)[-\max\{v_1 - b_{21}^r, 0\}] \cdot I_{b_{21}^r = b_{212}^r}}{\overline{\pi}_1^r - \underline{\pi}_1^r} \\ &= \frac{0 \cdot I_{b_{21}^r < b_{212}^r} + [-\max\{v_1 - b_{21}^r, 0\}] \cdot I_{b_{21}^r > b_{212}^r} + (1/2)[-\max\{v_1 - b_{21}^r, 0\}] \cdot I_{b_{21}^r = b_{212}^r}}{v_1 - \max\{r, v_2\} + \varepsilon}. \end{aligned} \tag{2.4}$$

Notice that in this framework, buyer 2 may collect a non-zero psychological payoff even when she loses, if she anticipates some counter-spite intent from buyer 1, i.e. $\delta_{21}^{r}(b_{21}^{r}, b_{212}^{r}) < 0$. Again, we set $\delta_{21}^{r}(b_{21}^{r}, b_{212}^{r}) = 0$ for r beyond v_1.

Buyer 2's utility at decision point r combines her monetary and psychological payoff, which is represented by a function $U_2^r : B_2^r \times B_1^r \times B_2^r \to \Re$, given by

$$U_2^r(b_2^r, b_{21}^r, b_{212}^r) = \pi_2(b_2^r, b_{21}^r) + \gamma_2 \cdot \hat{f}_{21}^{r}(b_{21}^r, b_{212}^r) \cdot f_2^r(b_2^r, b_{21}^r), \tag{2.5}$$

where γ_2 is a non-negative real number. The multiplicative form of the psychological payoff in the utility function (2.5) captures the reciprocal nature of spite. This is a common feature among the other existing reciprocal preference models cited above, each of which has its specific way of measuring the degree of intentions. Our utility model also follows the basic reciprocal preference model with the degree of intentions f_2^r and $\hat{f}_{21}^{r}$ constructed to be plausible and consistent with this auction context. Buyer 2 becomes a conventional economic agent maximizing a monetary payoff when either $\gamma_2 = 0$ or $f_2^r(b_2^r, b_{21}^r) = 0$.

Next consider the case of buyer 1. Suppose that buyer 1's first and second order beliefs at interim decision node r are $b_{12}^r \in B_2^r$ and $b_{121}^r \in B_1^r$. Let a function $U_1 : B_1^r \times B_2^r \times B_1^r \to \Re$ represent buyer 1's utility updated at the decision point r, defined in exactly the same manner as in the case of buyer 2, by

$$U_1^r(b_1^r, b_{12}^r, b_{121}^r) = \pi_1(b_1^r, b_{12}^r) + \gamma_1 \cdot \hat{f}_{12}^{r}(b_{12}^r, b_{121}^r) \cdot f_1^r(b_1^r, b_{12}^r), \tag{2.6}$$

where γ_1 is a non negative real number. The function f_1^r measures how spiteful buyer 1 is toward buyer 2, which is given by

$$\begin{aligned} f_1^r(b_1^r, b_{12}^r) &= \frac{\pi_2(b_{12}^r, b_1^r) - 0}{\overline{\pi}_2^r - \underline{\pi}_2^r} \\ &= \frac{\left\lfloor 0 \cdot I_{b_1^r > b_{12}^r} + (v_2 - b_1^r) \cdot I_{b_1^r < b_{12}^r} + (1/2)(v_2 - b_1^r) \cdot I_{b_1^r = b_{12}^r} \right\rfloor - 0}{v_1 - \max(r, v_2) + \varepsilon}. \end{aligned} \tag{2.7}$$

Its numerator is the distance between zero payoff that buyer 1 thinks buyer 2 should receive and the payoff that buyer 1's bid choice b_1^r makes possible for buyer 2, when buyer 1 assumes buyer 2 will bid b_{12}^r. This distance is measured against the payoff range that buyer 1 makes possible for buyer 2, when buyer 1 bids spitefully. Again, we set $f_1^r(b_1^r, b_{12}^r) = 0$ when $r \geq v_1 + \varepsilon$.

The index $\hat{f}_{12}^{r}$, measuring buyer 1's belief about how spiteful buyer 2 is toward buyer 1, consists of two components of the following form,

$$\hat{f}_{12}^{r}(b_{121}^{r}, b_{12}^{r}) = f_{12}^{r}(b_{121}^{r}, b_{12}^{r}) + \delta_{12}^{r}(b_{121}^{r}, b_{12}^{r}) . \tag{2.8}$$

The first component is given by

$$f_{12}^{r}(b_{12}^{r}, b_{121}^{r}) = \frac{\pi_1(b_{121}^{r}, b_{12}^{r}) - \hat{\pi}_1(b_{121}^{r}, b_{12}^{r})}{\overline{\pi}_1^{r} - \underline{\pi}_1^{r}}$$

$$= \frac{0 \cdot I_{b_{12}^{r} > b_{121}^{r}} + (v_2 - b_{12}^{r}) I_{b_{12}^{r} < b_{121}^{r}} + (1/2)(v_2 - b_{12}^{r}) \cdot I_{b_{12}^{r} = b_{121}^{r}}}{v_1 - \max(r, v_2) + \varepsilon} . \tag{2.9}$$

Its numerator is the distance between buyer 1's reference payoff and the payoff that buyer 1 believes buyer 2 makes available for buyer 1 by choosing bid b_{12}^{r} when in buyer 1's belief buyer 2 believes buyer 1 will bid b_{121}^{r}. The second component is given by

$$\delta_{12}^{r}(b_{12}^{r}, b_{121}^{r}) = \frac{0 \cdot I_{b_{12}^{r} < b_{121}^{r}} + [\min\{v_2 - b_{12}^{r}, 0\}] \cdot I_{b_{12}^{r} > b_{121}^{r}} + (1/2)[\min\{v_2 - b_{12}^{r}, 0\}] \cdot I_{b_{12}^{r} = b_{121}^{r}}}{\overline{\pi}_2 - \underline{\pi}_2}$$

$$= \frac{0 \cdot I_{b_{12}^{r} < b_{121}^{r}} + [\min\{v_2 - b_{12}^{r}, 0\}] \cdot I_{b_{12}^{r} > b_{121}^{r}} + (1/2)[\min\{v_2 - b_{12}^{r}, 0\}] \cdot I_{b_{12}^{r} = b_{121}^{r}}}{v_1 - \max(r, v_2) + \varepsilon} . \tag{2.10}$$

We set both f_{12}^{r} and δ_{12}^{r} to zero when $r \geq v_1 + \varepsilon$ .The second term in the numerator indicates how much buyer 2 is willing to risk in making her spiteful bid. As in the case of buyer 2, buyer 1 is a conventional agent with no psychological payoff when either $\gamma_1 = 0$ or $f_1^{r}(b_1^{r}, b_{12}^{r}) = 0$.

2.2 Equilibrium

In an ascending-bid auction, buyers make decisions multiple times before they arrive at a terminal node. In our model, the outset at $r = 0$ before the calling price starts rising directly corresponds to the decision situation in the second-price sealed-bid auction. Hence we interpret a bidding strategy $s_i(b_i^{0})$, $i \in \{1,2\}$ as a sealed bid b_i^{0} in the second-price auction. The rules of the ascending-bid auction do not allow buyers to reenter into bidding once they withdraw at some point, so at each interim decision point *r* buyers participate in a stage game where they play a second-price auction with a minimum selling price of *r*. It is useful in the later analysis to identify the property of each buyer's withdrawal bid b_i^{r} that is best response to b_j^{r} at an interim decision point r. For a given pair of beliefs at r, $b_{ij}^r \in B_j^r$ and $b_{iji}^r \in B_i^r$, $i, j \in \{1,2\}, i \neq j$, let $BR_i^r(b_{ij}^r, b_{iji}^r)$ denote a set of buyer i's best response of withdrawal bids such that

$$BR_i^r(b_{ij}^r, b_{iji}^r) = \left\{ b_i \in B_i^r \middle| U_i^r(b_i, b_{ij}^r, b_{iji}^r) \geq U_i^r(b_i', b_{ij}^r, b_{iji}^r), \forall b_i' \in B_i^r \right\}. \tag{2.11}$$

The beliefs held at r are said to be consistent with the decisions at r when $b_{ij}^r = b_j^r$ and $b_{iji}^r = b_i^r$, which we call an *interim consistency requirement.* Let us define an interim equilibrium at each r by considering optimal bidding strategies with consistent beliefs held at r when the buyers already arrive at the decision point r.

Definition 1 (Interim Equilibrium): A strategy profile $(s_1({}^*b_1^r), s_2({}^*b_2^r)) \in S_1 \times S_2$ is an interim equilibrium in an ascending-bid auction, if for each $i, j \in \{1,2\}, i \neq j$, ${}^*b_i^r \in BR_i^r(b_{ij}^r, b_{iji}^r)$ at a given calling price $r \in B$ with buyers' interim beliefs satisfying the interim consistency requirement, $b_{ij}^r = {}^*b_j^r$, and $b_{iji}^r = {}^*b_i^r$.

Let $E_{AB}^r \subset S_1 \times S_2$ denote the set of interim equilibrium strategy profiles, defined by $E_{AB}^r = \left\{ (s_1(b_1^r), s_2(b_2^r)) \in S_1 \times S_2 \middle| (b_1^r, b_2^r) \in B_1^r \times B_2^r, b_i^r \in BR_i^r(b_{ij}^r, b_{iji}^r), b_{ij}^r = b_j^r, b_{iji}^r = b_i^r, i \in \{1,2\}, i \neq j \right\}$.
While the interim equilibrium bidding strategy profiles at decision point r are optimal thereafter, strategy profiles exist that are optimal at those decision points before r but lead to termination then. Let us denote the set of such bidding strategy profiles that terminate the auction before r by $E_r = \left\{ (s_1, s_2) \middle| (s_1(b_1), s_2(b_2)) \in \bigcup_{k \in \{\varepsilon, \cdots, r\}} E_{AB}^{k-\varepsilon}, \text{and} \min\{b_1, b_2\} < r \right\}$. By the rules of the ascending-bid auction, if $E_{AB}^r = \emptyset$ for some $r \in B$, then $E_{AB}^{r'} = \emptyset$ for all $r' \in B$ such that $r' > r$. Let $\bar{r}$ denote the highest calling price possible where at least one buyer is active, defined by $\bar{r} \equiv \max \left\{ r \in B \middle| E_{AB}^r \neq \emptyset \right\}$.

Two notions of equilibrium are relevant for our intention-based ascending-bid auction model. One is an equilibrium analogous to Nash equilibrium in the conventional model, and the other is analogous to subgame perfect equilibrium. We call the former one a *weak intention-based equilibrium* and the latter a *strong intention-based equilibrium* as defined below.

Definition 2 (Weak Intention-based Equilibrium in ascending-bid auctions): A strategy profile

$(s_1(b_1^*), s_2(b_2^*)) \in S_1 \times S_2$ is a weak intention-based equilibrium in the ascending-bid auction, if for each $i, j \in \{1,2\}, i \neq j$, $(s_1(b_1^*), s_2(b_2^*)) \in E_{AB}^r \cup E_r$ at every $r \in \{0, \varepsilon, \cdots, \bar{r}\}$, under the consistency requirement on buyers' beliefs, $b_{ij}^0 = b_j^*$ and $b_{iji}^0 = b_i^*$.

Definition 3 (Strong Intention-based Equilibrium in ascending-bid auctions): A strategy profile $(s_1(b_1^*), s_2(b_2^*)) \in S_1 \times S_2$ is a strong intention-based equilibrium in the ascending-bid auction, if $(s_1(b_1^*), s_2(b_2^*)) \in E_{AB}^r$ at every $r \in \{0, \varepsilon, \cdots, \bar{r}\}$ for each $i, j \in \{1,2\}, i \neq j$.

Before characterizing the equilibrium, consider buyers' utility at some interim decision point *r*. Let us first examine buyer 2's utility. Given that she arrives at some *r*, her first and second order beliefs must be $b_{21}^r \geq r$ and $b_{212}^r \geq r$. Suppose that her beliefs satisfy the interim consistency requirement $b_{21}^r = b_1^r$ and $b_{212}^r = b_2^r$. If buyer 2 chooses to withdraw before her opponent $b_2^r < b_{21}^r$, then her losing utility is given by

$$U_2^r(b_2^r, b_{21}^r, b_{212}^r) = \gamma_2 \left(\frac{v_2 - b_2^r}{v_1 - \max\{r, v_2\} + \varepsilon} \right) \left(\frac{-\max\{v_1 - b_{21}^r, 0\}}{v_1 - \max\{r, v_2\} + \varepsilon} \right) \tag{2.12}$$

when $r \leq v_1$, and $U_2^r(b_2^r, b_{21}^r, b_{212}^r) = 0$, otherwise. If buyer 2 chooses to stay in the auction beyond b_{21}^r, she will collect the winning payoff of

$$U_2^r(b_2^r, b_{21}^r, b_{212}^r) = v_2 - b_{21}^r. \tag{2.13}$$

If buyer 2 places a tie bid, $b_2^r = b_{21}^r = b_{212}^r$, then her utility is

$$U_2^r(b_2^r, b_{21}^r, b_{212}^r) = (1/2)(v_2 - b_{21}^r) + (1/2)\gamma_2 \left(\frac{v_2 - b_2^r}{v_1 - \max\{r, v_2\} + \varepsilon} \right) \left(\frac{-\max\{v_1 - b_{21}^r, 0\}}{v_1 - \max\{r, v_2\} + \varepsilon} \right) \tag{2.14}$$

when $r \leq v_1$ holds, and otherwise

$$U_2^r(b_2^r, b_{21}^r, b_{212}^r) = (1/2)(v_2 - b_{21}^r). \tag{2.15}$$

For example, when buyer 2 is already at the decision point $\hat{r} \in \{v_2 + \varepsilon, \cdots, v_1 - \varepsilon\}$, and if her anticipated bid of buyer 1 is $b_1^{\hat{r}} = b_{21}^{\hat{r}} = \hat{b} \in \{v_2 + \varepsilon, \cdots, v_1 - \varepsilon\}$, it is easy to check that buyer 2's losing utility is increasing in her own bid, and her winning utility is constant. This is buyer 2's spite region. If the parameters are such that her losing utility exceeds her winning utility, her best response among those early withdrawal strategies is to bid ε below her opponent's expected withdrawal level, $b_2^r = \hat{b} - \varepsilon$.

Next consider the case of buyer 1 facing a decision problem at *r*. She must decide when to drop out, that is, to stay until b_1^r, under her first order belief b_{12}^r and her second order belief b_{121}^r. Suppose again that these beliefs satisfy the interim consistency requirement, $b_2^r = b_{12}^r$ and $b_1^r = b_{121}^r$, and logical consistency b_2^r, b_{12}^r, $b_1^r, b_{121}^r \geq r$. If buyer 1 decides to stay in longer than her opponent, i.e., $b_1^r > b_{12}^r$, she expects to receive winning utility of

$$U_1^r(b_1^r, b_{12}^r, b_{121}^r) = v_1 - b_{12}^r. \tag{2.16}$$

If buyer 1 plans to withdraw earlier than her opponent, $b_1^r < b_{12}^r$, then her losing utility is given by

$$U_1^r(b_1^r, b_{12}^r, b_{121}^r) = \gamma_1 \left(\frac{v_2 - b_1^r}{v_1 - \max\{r, v_2\} + \varepsilon} \right) \left(\frac{\min\{(v_2 - b_{12}^r), 0\}}{v_1 - \max\{r, v_2\} + \varepsilon} \right), \tag{2.17}$$

when $r \leq v_1$, and her losing utility is zero otherwise. Placing a tie bid of $b_1^r = b_{12}^r$ brings her utility equal to one half of (2.16) and one half of (2.17).

No essential difference exists between the two buyers' payoff structure at an interim decision point when their beliefs satisfy the interim consistency requirement. For example, consider buyer 1 at the decision point $\hat{r} \in \{v_2 + \varepsilon, \cdots, v_1 - \varepsilon\}$. If buyer 1 expects buyer 2 to plan a withdrawal bid at $b_2^{\hat{r}} = b_{12}^{\hat{r}} = \hat{b} \in \{v_2 + \varepsilon, \cdots, v_1 - \varepsilon\}$, the case of $b_1^{\hat{r}} < \hat{b}$ is buyer 1's counter spite region where her losing utility increases in her bid.

Since both buyers' losing payoff is increasing in their own bids, their decision should be based on a comparison between the maximum losing payoff and the winning payoff. Then, if there is a tie bid that equates the losing payoff and winning payoff, it is the *threshold bid* that divides buyers' decision to stay in or drop out. In order to identify such a threshold bid, let us define following two functions $\varphi_1 : \overline{B} \times \overline{B} \to \overline{B}$ and $\varphi_2 : \overline{B} \times \overline{B} \to \overline{B}$, where $\overline{B} = [0, \overline{b}] \subset \Re$ which is a convex closure of discrete *B*. It is convenient to consider $\overline{B}$ first and add the restriction of a minimum bid unit later.

$$\varphi_1(x,r)=\begin{cases} v_1-x, \text{if } r>v_1 \\ [v_1-x]-(v_2-x)\left(\dfrac{\gamma_1 \min(v_2-x,0)}{(v_1-\max\{v_2,r\}+\varepsilon)^2}\right), \text{otherwise.}\end{cases} \tag{2.18}$$

$$\varphi_2(x,r)=\begin{cases} (v_2-x), \text{if either } x\geq v_1 \text{ or } r\geq v_1, \\ (v_2-x)-(v_2-x)\left(\dfrac{\gamma_2(-\max(v_1-x,0))}{(v_1-\max(r,v_2)+\varepsilon)^2}\right), \text{otherwise}\end{cases} \tag{2.19}$$

Assume that buyers' beliefs satisfy the interim consistency requirement. Equation (2.18) represents the difference between buyer 1's winning payoff (2.16) and losing payoff (2.17) when $b_1^r=b_{12}^r=x$. Similarly, equation (2.19) is the difference between payoff (2.13) and (2.14) of buyer 2 when $b_2^r=b_{21}^r=x$. We can thus identify their threshold bids as a solution of $\varphi_i(x,r)=0$, $i\in\{1,2\}$. Let us denote this solution by β_i^r such that $\varphi_i(\beta_i^r,r)=0$. In particular, denote buyer 1's threshold value at $r^*\in\overline{B}$ by β_H, that is, $\beta_1^{r^*}=\beta_H$, when $b_1^{r^*}=b_{12}^{r^*}=x=r^*$.

After identifying $\beta_i^r\in\overline{B}$ (and $\beta_H\in\overline{B}$) and its behavior for each relevant $r\in\overline{B}$, we find an integer $\hat{\beta}_i^r$ (and $\hat{\beta}_H$) such that $\beta_i^r\in[\hat{\beta}_i^r\varepsilon,(\hat{\beta}_i^r+1)\varepsilon)$ (and $\beta_H\in[\hat{\beta}_H\varepsilon,(\hat{\beta}_H+1)\varepsilon)$) for all relevant $r\in B$ to be consistent with a minimum bid unit ε.

Lemma 1a: There exists a threshold bid $\beta_2^r\in\overline{B}$ only when $r\leq v_1$, and $\beta_2^r=v_2$ for all such relevant *r*.

Lemma 1b: (i) There exists a unique threshold bid $\beta_1^r\in\overline{B}$ only when $r\in[0,v_1]$.

(ii) For all $r\in[0,v_2]$, $\beta_1^r=\beta_1^0$.

(iii) For all $r\in(v_2,\beta_H]$, $\beta_1^r\in(v_2,v_1)$ and $\beta_1^r\geq r$.

(iv) β_1^r is strictly decreasing in *r*, for all $r\in(v_2,\beta_H]$.

(v) $\beta_1^{r=\beta_H}=\beta_H$.

(vi) For all $r\in(\beta_H,\overline{b}]$, $\beta_1^r<r$.

(vii) β_1^r is strictly decreasing in γ_1, for all $r\in[0,\beta_H]$

Lemma 1c: (i) For all $r\in\{0,\varepsilon,\cdots,v_2-\varepsilon,v_2\}$, $\hat{\beta}_1^r=\hat{\beta}_1^0$.

(ii) For all $r\in\{v_2+\varepsilon,\ v_2+2\varepsilon,\cdots,\hat{\beta}_H\varepsilon\}$, $\hat{\beta}_1^r\varepsilon\geq r$.

(iii) $\hat{\beta}_1^r$ is non-increasing in *r*, for all $r \in \{v_2 + \varepsilon,\ v_2 + 2\varepsilon, \cdots, \hat{\beta}_H \varepsilon\}$.

(iv) For all $r \in \{(\hat{\beta}_H + 1)\varepsilon, \cdots, v_1\}$, $\hat{\beta}_1^r \varepsilon < r$.

(v) $\hat{\beta}_1^r$ is non-increasing in γ_1, for all $r \in \{0, \varepsilon, \cdots, \hat{\beta}_H \varepsilon\}$

When $v_1 \le r$, it is obvious from (2.18) and (2.19) that both buyers always prefer to lose. From lemma 1a, we can tell that when $r \le v_1$, buyer 2 prefers to place a higher bid if $b_{21}^r \le v_2$, and prefers to place a maximum losing bid $b_2^r \le b_{21}^r - \varepsilon$ otherwise. Lemma 1c (iv) implies that buyer 1 has no meaningful threshold bid when the calling price falls in the range $r \in \{(\hat{\beta}_H + 1)\varepsilon, \cdots, \bar{b}\}$, because planning to withdraw at $\hat{\beta}_1^r \varepsilon < r$ is not possible when buyer 1 has already arrived at decision point *r*. Thus, roughly put, at every interim decision point such that $r \in \{v_2 + \varepsilon, v_2 + 2\varepsilon, \cdots, \hat{\beta}_H \varepsilon\}$, buyer 1 prefers to lose if her anticipated opponent's withdrawal bid b_{12}^r exceeds the threshold value of β_1^r by making a withdrawal bid equal to $b_{12}^r - \varepsilon$, and prefers to win otherwise. But at each decision point with $r \in \{(\hat{\beta}_H + 1)\varepsilon, \cdots, \bar{b}\}$, buyer 1 always prefers to lose. Proposition 1 below identifies interim equilibrium bid strategy profiles for each relevant *r*, where buyer 1 is always a winner and buyer 2 is a loser by planning to withdraw ε before buyer 1's expected withdrawal bid level.

Proposition 1: (i) Under the interim consistency requirement, for all $r \in B$ such that $r \le \bar{r}$, $(s_1(b_1^r), s_2(b_2^r)) \in E_{AB}^r$ if and only if $b_2^r = b_1^r - \varepsilon$, where $b_1^r \in B_1^r$ satisfies $\max\{r, v_2\} + \varepsilon \le b_1^r \le \hat{\beta}_1^r \varepsilon$, when $\beta_1^r = \hat{\beta}_1^r \varepsilon$, and $\max\{r, v_2\} + \varepsilon \le b_1^r \le [\hat{\beta}_1^r + 1]\varepsilon$, when $\beta_1^r \ne \hat{\beta}_1^r \varepsilon$. (For all $r \ge \bar{r} + \varepsilon$, $E_{AB}^r = \emptyset$.) (ii) The ultimate terminal node $\bar{r}$ is given by $\bar{r} = (\hat{\beta}_H - 1)\varepsilon$ if $\hat{\beta}_H \varepsilon = \beta_H$, and $\bar{r} = \hat{\beta}_H \varepsilon$, if $\hat{\beta}_H \varepsilon \ne \beta_H$.

The next lemma is a direct consequence of Lemma 1a,b,c, and Proposition 1, which characterizes a relation among sets of interim equilibrium strategy profiles.

Lemma 2: $E_{AB}^{\bar{r}} \subset E_{AB}^{r-\varepsilon} \subset \cdots \subset E_{AB}^{v_2} = \cdots = E_{AB}^{\varepsilon} = E_{AB}^0$.

Recall that an interim equilibrium set E^r_{AB} contains only those bid strategies with a withdrawal bid equal to the current calling price *r* or higher. Note also from Lemma 2 that E^r_{AB} when $r \le \bar{r} - \varepsilon$ contains strategy pairs that are never reached at the ultimate terminal node $\bar{r}$. This means that some beliefs consistent in one interim equilibrium set may not be consistent in another interim equilibrium set. Let E^w_{AB} and E^s_{AB} denote the set of weak and strong intention-based equilibrium bidding strategies in an ascending-bid auction, respectively. Proposition 2 shows that both sets of intention-based equilibrium bidding strategies of an ascending-bid auction are determined mainly by the smallest interim equilibrium set $E^{\bar{r}}_{AB}$.

Proposition 2:

(i) A strategy profile $(s_1(b_1^*), s_2(b_2^*)) \in E^w_{AB}$ if and only if $(s_1(b_1^*), s_2(b_2^*)) \in E^{\bar{r}}_{AB} \cup E_{\bar{r}}$.

(ii) A strategy profile $(s_1(b_1^*), s_2(b_2^*)) \in E^s_{AB}$ if and only if $(s_1(b_1^*), s_2(b_2^*)) \in E^{\bar{r}}_{AB}$.

Let E_{SP} denote a set of equilibrium strategy profiles in a second-price auction. Since the initial stage of an ascending-bid auction, $r = 0$, is exactly the same as the second-price auction, $E_{SP} = E^0_{AB}$. We therefore have an ordering inclusion relationship among E^w_{AB}, E_{SP}, and $\{E^r_{AB}\}$, summarized in Corollary below.

Corollary: (i) $\hat{\beta}(0) \ge \hat{\beta}(\bar{r})$. (ii) For all $r \in \{\varepsilon, \cdots, \bar{r}\}$, $E^w_{AB} \subseteq E^r_{AB} \cup E_r \subseteq E_{SP}$.

Let us point out several key observations about the equilibrium set. First, our model contains the conventional model, where the spite motive plays no role, as a special case where either $\gamma_i = 0$ or $f_i = 0$ for $i \in \{1,2\}$. Let us denote the set of Nash equilibrium strategy profiles in the conventional second-price auction by $E^c_{SP} \subset S_1 \times S_2$, and the conventional ascending-bid auction by $E^c_{AB} \subset S_1 \times S_2$, for two buyers with $v_1 > v_2$. E^c_{SP} can be described as

$$E^c_{SP} = \left\{ (s_1(b_1), s_2(b_2)) \in S_1 \times S_2 \,\middle|\, (b_1, b_2) \in B_1 \times B_2, \text{and} \begin{array}{ll} b_2 \ge v_1 + \varepsilon, & \text{if } b_1 \le v_2 \\ b_2 \ge b_1 - \varepsilon & \text{if } v_2 + \varepsilon \le b_1 \le v_2 \\ b_2 \le v_1 & \text{if } b_1 \ge v_1 + \varepsilon. \end{array} \right\}.$$

E_{SP}^c contains multiple Nash equilibria that are both efficient and inefficient, and in particular the efficient equilibrium set includes the value-revealing dominant strategy equilibrium $(b_1', b_2') = (v_1, v_2)$. Figure 1 shows an example, when $v_1 = 800$, $v_2 = 700$, and $\varepsilon = 10$.

Applying our interim equilibrium notion to the conventional model, it is immediate that

$$\left(E_{AB}^c\right)^r = \left\{ (s_1(b_1^r), s_2(b_2^r)) \in S_1 \times S_2 \middle| (b_1^r, b_2^r) \in B_1^r \times B_2^r, \text{ and } \begin{array}{ll} b_2^r \geq v_1 + \varepsilon, & \text{if } b_1^r \leq v_2 \\ b_2^r \geq b_1 - \varepsilon & \text{if } v_2 + \varepsilon \leq b_1^r \leq v_2 \\ b_2^r \leq v_1 & \text{if } b_1^r \geq v_1 + \varepsilon. \end{array} \right\}$$

for all $r \in \{0, \varepsilon, \cdots, v_2 - \varepsilon, v_2\}$, and

$$\left(E_{AB}^c\right)^r = \left\{ (s_1(b_1^r), s_2(b_2^r)) \in S_1 \times S_2 \middle| (b_1^r, b_2^r) \in B_1^r \times B_2^r, \text{ and } \begin{array}{ll} b_2^r \geq b_1^r - \varepsilon, & \text{if } b_1^r \leq v_1 \\ b_2^r \leq v_1 & \text{if } b_1^r \geq v_1 + \varepsilon. \end{array} \right\}$$

for all $r \in \{v_2 + \varepsilon,\ v_2 + 2\varepsilon, \cdots, \bar{b}\}$. Therefore, for buyers without spite or counter-spite motives, the set of Nash equilibrium bidding strategies in the second-price auction coincides with the bidding strategy set of our intention-based equilibrium, which is no surprise since there is no room for intentions in the conventional model. Consequently, the equilibrium set of the ascending-bid auction and second-price auctions for buyers without reciprocal spite motives are identical, which corresponds to the well known isomorphism between the two auction mechanisms.

Our second observation is that introducing the reciprocal spite motivation makes E_{SP} much smaller than E_{SP}^c. Figure 2 shows an example of withdrawal bid profiles (b_1, b_2) that satisfies $E_{SP} = E_{AB}^0 = \left\{ (s_1(b), s_2(b - \varepsilon)) \middle| v_2 + \varepsilon \leq b \leq \hat{\beta}_1^0 \varepsilon \right\}$ when $v_1 = 800$, $v_2 = 700$, $\gamma_1 = 5$, $\varepsilon = 10$. In this case, buyer 1's threshold bid at $r = 0$ is calculated as $\hat{\beta}_1^0 \varepsilon = 790$, and the set of equilibrium bid profiles is illustrated by the dots located within a line segment parallel to the 45 degree line with lower bound of $(710,700)$ and upper bound of $(790,780)$. As compared to Figure 1, the equilibrium set in Figure 2 does not contain any inefficient withdrawal bid profiles as well as those bid profiles located below 45 degree line by distance of 2ε or more. The intuition behind this is simple. Consider a strategy profile $(s_1(\tilde{b}_1), s_2(\tilde{b}_2)) \in E_{SP}^c$ such that $\tilde{b}_1 < \tilde{b}_2$, whose withdrawal bids $(\tilde{b}_1, \tilde{b}_2)$ are located in the upper left inefficient equilibrium set in Figure 1 where $v_1 < \tilde{b}_2$. In the conventional model without spite and counter-spite motivations, buyer 1 receives a zero monetary payoff by placing any bid less than $\tilde{b}_2$. But with spiteful preferences, buyer 1 has an incentive to increase her bid up to $\tilde{b}_2 - \varepsilon$. Buyer 2's original bid $\tilde{b}_2$, however, is no longer a best response to buyer 1's bid of $\tilde{b}_2 - \varepsilon$. Buyer 2 would instead prefer to lose by placing a lower but spiteful bid. In this way, every inefficient Nash equilibrium point in E_{SP}^{c} is not compatible with our weak intention-based equilibrium notion with reciprocal spite motives. A similar intuition applies to explain why E_{SP} does not contain $\{(b_1, b_2) \in B_1 \times B_2 | b_2 \le b_1 - 2\varepsilon, b_1 \ge v_2 + \varepsilon\}$. Thus, the value-revealing withdrawal bids $(b_1', b_2') = (v_1, v_2)$ no longer generate a weak intention-based equilibrium bidding strategy profile, unless $v_1 = v_2 + \varepsilon$. However, each set $\left(E_{AB}^{r} \cup E_r\right)$ for all $r \in \{0, \varepsilon, \cdots, \bar{r}\}$ keeps a strategy profile $(s_1(\hat{b}_1), s_2(\hat{b}_2)) = (s_1(v_2 + \varepsilon), s_2(v_2))$ which generates the same allocation outcome as $(s_1(b_1'), s_2(b_2')) = (s_1(v_1), s_2(v_2))$ as a lower boundary.

Our third observation concerns the dynamic nature of an ascending-bid auction specific to our intention-based model. The dotted line between $(b_1, b_2) = (v_2, v_2 + \varepsilon)$ and $(b_1, b_2) = ((\hat{\beta}_H - 1)\varepsilon, \hat{\beta}_H \varepsilon)$ in Figure 2 is an example of the withdrawal bids that correspond to the set E_{AB}^{w}, when $v_1 = 800$, $v_2 = 700$, $\gamma_1 = 5$, and $\varepsilon = 10$. In this case, we find $\hat{\beta}_H^{r} \varepsilon = 770$ which is strictly lower than the upper bound $\hat{\beta}_1^{0} \varepsilon = 790$ of $E_{SP} = E_{AB}^{0}$. Thus, E_{AB}^{w} is a proper subset of E_{SP}. In an ascending-bid auction, once the calling price goes beyond v_2, buyers start revising their psychological payoff components of their utilities. Observing the payoff possibilities shrink as the calling price rises, both buyers add weight to their own spitefulness. Buyer 2 experiences a greater joy from shading her opponent's winning monetary payoff, which at the same time, makes buyer 1 less tolerant and prompts her to resort to an earlier retaliatory withdrawal. Knowing this effect on buyer 1, buyer 2 would not push her luck by staying longer in the auction. As a result, the upper bound of each interim equilibrium set starts descending.

These psychological dynamics are different in nature from the issue of Nash equilibrium versus subgame perfect Nash equilibrium. In the conventional model, if we consider a set of subgame perfect equilibrium strategy profiles for the ascending-bid auction, the equilibrium set consists of those strategy profiles that survive at the ultimate withdrawal decision point $\bar{r} = v_1$ for buyer 1. The withdrawal bids for the subgame perfect equilibrium strategy profiles are

$\{(b_1, b_2) | v_2 \le b_2 \le v_1 - \varepsilon, b_1 = v_1\}$, which consists of only efficient profiles and are limited to those with $b_1 = v_1$. The equilibrium set is certainly much smaller than E_{SP}^{c} due to the refinement. When we consider the strong intention-based equilibrium of Definition 3 for the ascending-bid auction with reciprocal spite motives, the withdrawal bid for E_{AB}^{s} is given by $\{(b_1, b_2) | b_2 = (\hat{\beta}_H - 1)\varepsilon, b_1 = \hat{\beta}_H \varepsilon\}$. Thus, the set E_{AB}^{s} is a singleton and coincides with the upper bound of E_{AB}^{w}. Again, the value-revealing withdrawal bids $(b_1', b_2') = (v_1, v_2)$ do of course remain dominant in the subgame perfect equilibrium in the conventional model. Thus the second-price auction and the ascending-bid auction are equivalent as long as buyers employ the dominant strategy, but this strategy is neither dominant nor equilibrium in our intention-based model. Yet in E_{AB}^{s} the psychological dynamics again restrict buyer 2 from bold over bidding beyond $b_2 = (\hat{\beta}_H - 1)\varepsilon$, and induce buyer 1 to retaliate by withdrawing at $b_2 = \hat{\beta}_H \varepsilon$, earlier than v_1. Therefore, in both cases of weak and strong intention-based equilibrium, the psychological dynamics prevent higher range prices from arising in the ascending-bid auction. This provides a basis for our main testable hypotheses in Section 4 that the ascending-bid auction makes buyer 2 overbid less boldly and consequently generates lower prices on average in the ascending-bid than the second-price auction.

Lastly, none of the preceding three basic observations depends on $\gamma_i \neq 0$, $i \in \{1,2\}$. The coefficient γ_i reflects the magnitude of the reciprocal spite intention. The only effect of γ_i we have observed is that each β_1^r is strictly decreasing in γ_1 for all r beyond v_2, as stated in Lemma 1b. It is quite intuitive that an increase in γ_1 enhances the effect of psychological dynamics, in the sense that it makes buyer 2's overbid more risky in the face of a greater threat of counter-spite under-bid by buyer 1. Consequently, the ascending-bid auction lets buyers prevent more of their consumer surplus from being acquired by the monopolistic seller. Together with the third observation, we may say that the dynamics of the ascending-bid auction format makes use of spiteful human nature to allow buyers to achieve better market outcomes than the second-price auction.

2.3. Unknown Values (Incomplete Information)

This subsection considers the case where each individual buyer does not know her opponent's value but does know its probability distribution. This is the incomplete information environment commonly adopted in the auction literature. We remove the restriction of a minimum bid unit and carry out the analysis in the continuous environment, which is also common. In the preceding two subsections with the complete information environment, we adopted the discrete setting because we needed to have the bidding strategy set finite in order to ensure the equilibrium to exist both in the conventional model and our intention-based model. None of the basic properties we derived there, however, critically depend on the discreteness of the environment. Retaining that discrete environment in this subsection would merely complicate the analysis and add no essential implications.

Let $\overline{V} = [\underline{v}, \overline{v}] \subset \Re_+ \cup \{0\}$ be an interval from which each bidder's value is drawn. Each buyer knows a common prior distribution of private values of buyers including hers. Once she knows her own private value she constructs a conditional probability distribution over her opponent's value, which is denoted by a cumulative distribution function $G : \overline{V} \to [0,1]$ with density function $g : \overline{V} \to [0,1]$.

Applying minor modification to the utility with reciprocal spite constructed in preceding subsections, buyer *i*'s utility with $v_i > v_j$ can be described by

$$U_i(b_i, b_{ij}, b_{iji}) = \pi_i(b_i, b_{ij}) + \gamma_i f_i(b_i, b_{ij}) \cdot \hat{f}_{ij}(b_{ij}, b_{iji}) . \qquad (2.20)$$

The first term in (2.20) represents her monetary payoff, which is given by

$$\pi_i(b_i, b_{ij}) = (v_i - b_{ij}) I_{b_i > b_{ij}} + 0 \cdot I_{b_i < b_{ij}} + (1/2)(v_i - b_j) I_{b_i = b_{ij}} . \qquad (2.21)$$

The second term of (2.20) represents her psychological payoff. The index f_i reflects her own spitefulness toward buyer *j*, which takes the form of

$$f_i(b_i^r, b_{ij}^r) = \frac{\pi_j(b_{ij}^r, b_i^r) - \max\{(v_i - v_j), 0\}}{\overline{\pi}_j^r - \underline{\pi}_j^r}$$

$$= \frac{[(v_j - b_i) - \max\{(v_i - v_j), 0\}] I_{b_{ij}^r > b_i} + 0 \cdot I_{b_{ij}^r < b_i} + (1/2)[(v_j - b_i) - \max\{(v_i - v_j), 0\}] I_{b_{ij}^r = b_i}]}{v_i - \max(r, v_j)} , \qquad (2.22)$$

for $r \in [0, v_i)$, and $f_i(b_i^r, b_{ij}^r) = 0$ for $r \in [v_i, \overline{v}]$. The other index $\hat{f}_{ij}$ reflects buyer *i*'s expected damage by buyer *j*, having two components $\hat{f}_{ij} = f_{ij} + \delta_{ij}$. The first component is a direct consequence of *j*'s bid (in *i*'s expectation), given by

$$f_{ij}(b_{iji}^r, b_{ij}^r) = \frac{\pi_i(b_{iji}^r, b_{ij}^r) - \max\{(v_i - v_j), 0\}}{\overline{\pi}_i^r - \underline{\pi}_i^r}$$

$$= \frac{0 \cdot I_{b_{ij}^r > b_{iji}^r} + \left[(v_i - b_{ij}^r) - \max\{v_i - v_j), 0\}\right] I_{b_{ij}^r < b_{iji}^r} + \left[(1/2)(v_i - b_{ij}^r) - \max\{v_i - v_j), 0\}\right] I_{b_{ij}^r = b_{iji}^r}}{v_i - \max(r, v_j)}, \quad (2.23)$$

for $r \in [0, v_i)$, and $f_{ij}(b_{iji}^r, b_{ij}^r) = 0$ for $r \in [v_i, \overline{v}]$. The second component of $\hat{f}_{ij}$ is consequence of *j*'s intention reflected in *j*'s bid choice in *i*'s view, defined for $r \in [0, v_i)$ by

$$\delta_{ij}(b_{iji}^r, b_{ij}^r) = \frac{0 \cdot I_{b_{ij}^r < b_{iji}^r} + \rho_{ij}(b_{iji}^r, b_{ij}^r) \cdot I_{b_{ij}^r > b_{iji}^r} + (1/2)\rho_{ij}(b_{iji}^r, b_{ij}^r) \cdot I_{b_{ij}^r = b_{iji}^r}}{\overline{\pi}_j^r - \underline{\pi}_j^r}$$

$$= \frac{0 \cdot I_{b_{ij}^r < b_{iji}^r} + \rho_{ij}(b_{iji}^r, b_{ij}^r) \cdot I_{b_{ij}^r > b_{iji}^r} + (1/2)\rho_{ij}(b_{iji}^r, b_{ij}^r) \cdot I_{b_{ij}^r = b_{iji}^r}}{v_i - \max(r, v_j)}, \quad (2.24)$$

where

$$\rho_{ij}(b_{iji}^r, b_{ij}^r) = \begin{cases} \min\{(v_j - b_{ij}^r), 0\}, \text{ if } v_i \geq v_j, \\ -\max\{(v_j - b_{ij}^r), 0\}, \text{ if } v_i < v_j. \end{cases} \quad (2.25)$$

And $\delta_{ij}(b_{iji}^r, b_{ij}^r) = 0$ for $r \in [v_i, \overline{v}]$.

Since we have only two buyers, we omit the subscript indicating a buyer. We consider a continuous and continuously differentiable bidding function $b_r : \overline{V} \to \overline{B}$, with $b_r(0) = 0$ at every *r*. This is a withdrawal bid plan at *r*. Let $z \in \overline{V}$ denote a buyer's opponent's value, which she perceives as a random variable following a cumulative probability distribution function $G_v : \overline{V} \to [0,1]$ with a density function $g_v : \overline{V} \to [0,1]$, obtained from value distribution function G conditional on her own value v. Based on the same bid function $b_r(\cdot)$, the buyer can estimate the distribution of her opponent's bid. Since the calling price has already climbed up to *r*, the probability distribution of her opponent's bid must be conditional on that since the possibility of her opponent's bid being less than *r* has already been eliminated. Since we are interested in a symmetric equilibrium bid function, $b_r(\cdot)$ must be strictly increasing in its argument. Then, if a buyer makes a withdrawal bid plan corresponding to a value level x, i.e. $b_r(x)$, she wins when her opponent's value falls below x, i.e., $z < x$, and she loses otherwise.[4] Note that the value x must be such that $b_r(x) \in \overline{B} \setminus [0, r)$ at r, so that $x \geq b_r^{-1}(r)$, otherwise the buyer has already retired the bidding before r. Given consistent beliefs, the expected payoff of a buyer when she submits a withdrawal plan $b_r(x)$ at a decision point r, is given by;

$$EU^r(x, v) = \int_{b_r^{-1}(r)}^{x} (v - b_r(z))\, dG_v(z) \Big/ \left(1 - G_v(b_r^{-1}(r))\right)$$

$$+ \int_x^{\overline{v}} \left(\frac{z - b_r(x) - \max\{v - z, 0\}}{D^r(v, z)} \right) \gamma \left(\frac{\rho^r(v, z)}{D^r(v, z)} \right) dG_v(z) / \left(1 - G_v(b_r^{-1}(r))\right), \quad (2.26)$$

where $b_{ij}^r = b_r(x)$ and $b_{iji}^r = b_r(x)$, and

$$\rho^r(v, z) = \begin{cases} \min\{z - b_r(z)\}, \text{if } v \geq z \\ -\max\{z - b_r(z), 0\}, \text{if } v < z \end{cases},$$

$$D^r(v, z) = \begin{cases} |v - \max\{r, z\}|, \text{if } r \in [0, \max\{v, z\}) \\ 0, \quad\quad \text{if } r \in [\max\{v, z\}, \overline{v}] \end{cases}.$$

We define that the inside of the parentheses of the second term of (2.26) vanishes when the nominator D^r is zero.

Each buyer maximizes her EU^r by choosing an optimal withdrawal bid plan x at every relevant decision point r. Analogous to the interim equilibrium notion of Definition 2 in Section 2, we can think of a symmetric interim equilibrium generated by $b_r(x)$ that maximizes (2.26) for every buyer, equivalently for every $v \in \overline{V}$ when $x = v$, at given r. Then, a symmetric equilibrium in the second-price auction can be regarded as a symmetric interim equilibrium when $r = 0$. On the other hand, in the ascending-bid auction the ultimate withdrawal decision should satisfy $b_r(v) = r$. Thus, a symmetric equilibrium bidding function in the ascending-bid auction should satisfy the conditions for a symmetric interim equilibrium as well as $b_r(v) = r$. The next proposition identifies a value-revealing bidding function as a unique symmetric equilibrium strategy.

[4] Since we consider only continuous bid function and continuous value distribution, the probability of a tie is zero.

Proposition 3: (i) There exists a unique symmetric equilibrium strategy where each buyer plans to withdraw at her own value, at every interim equilibrium. (ii) In an ascending-bid auction as well as in a second-price sealed bid auction, there exists a unique symmetric equilibrium strategy where each buyer plans to withdraw at her own value.

It was the reciprocal spite motive that eliminated the value-revealing bidding strategy from intention-based equilibrium in the complete information setting of Section 2.2. In the incomplete information setting, even when buyers are still motivated with reciprocal spite a value-revealing bidding function remains an equilibrium. This is also the case in the conventional setting with no psychological payoff. A key difference, however, is that a value-revealing bidding function constitutes a dominant strategy equilibrium in the conventional model, but not in the model with spiteful preferences.

The intuition behind Proposition 3 is as follows. When buyers face value uncertainty regarding their opponents, they are unable to identify their relative value position, which is a main factor igniting their spiteful and counter spiteful reactions. This part is embedded in the second term in (2.26). Because they are not certain of their relative position, they lose a basis of either overbidding or underbidding. This is the driving force leading to this equilibrium result.

3. Experimental Design

The experiment consisted of 7 sessions of 12 subjects each (84 total subjects), all conducted with undergraduate econ major students at Shinshu University. Subjects bid in a series of two-bidder auctions with one item for sale. The principal treatment variables were the auction format (ascending price versus second-price sealed-bid) and information conditions (complete versus incomplete). Both of these treatment variables were varied within sessions, and in 4 sessions all subjects bid in both formats and both information conditions. In the remaining 3 sessions subjects only bid in complete information, sealed-bid auctions. Subjects submitted bids for 6 to 10 consecutive periods within each treatment configuration. A secondary treatment variable was the matching rule. This was also varied within sessions, so sometimes subjects bid against the same opponent for 6 to 10 periods, and at other times subjects bid against randomly-changing opponents every period. The matching rule was common knowledge. The order of both the principal and secondary treatment variables changed across sessions.

In the complete information treatment, the two possible resale values for the two bidders were 700 and 800 yen. These two values were randomly assigned each period, and this was common knowledge. Therefore, after a bidder learned that her resale value was 800 yen, for example, she knew with certainty that the other bidder's resale value was 700 yen. In the incomplete information treatment, resale values were drawn independently for each bidder each period from the discrete uniform distribution between 500 and 800 yen. This probability distribution was common knowledge, but individuals only learned their own value draw. Bids were constrained to 10-yen increments, but value draws could be any whole yen amount in the feasible range. In all ascending-bid auction treatments the clock price increased in 10-yen increments.

Subjects received the difference between their resale value and their price paid when they won the auction. The price was determined by the lowest bid or the first drop-out price, depending on the auction format, with the highest or the remaining bidder winning the auction. (Ties were resolved randomly.) Subjects received written instructions to describe the auction rules and procedures, which they first read in silence before the experimenter read them aloud. A translation of the instructions is shown in Appendix B. At the conclusion of the session subjects received their cumulative auction winnings in cash, along with a 1000 yen show-up payment. Payments (including this show-up payment) averaged about 4500 yen, and ranged between 1590 and 10788 yen. Sessions typically lasted about 150 minutes.

4. Experimental Results

4.1 Overview

Recall that in the complete information environment, the valuations are either 700 or 800 yen. Figures 3 and 4 display the frequency distribution of bids for the low-value (700) and high-value (800) bidder, respectively.[5] In the ascending-bid auction, 30 of the 291 bids for the low value bidder are not observed directly, since the low-value bidder won the auction when the

[5] In the data analysis we exclude "throw-away" and overtly collusive bids, defined as bids less than 200, because our interest is in serious bids and competing buyers. A total of 67 bids were excluded using this criterion, which represents about two percent of the 3310 total bidding opportunities. We found only limited evidence that behavior differed between fixed groups and randomly-reformed groups of bidders. Collusion attempts were more common and successful in fixed groups, but since we are excluding collusive bids from our analysis the behavior is typically not significantly different across matching rules for the non-collusive bidders. Therefore, this initial summary and some of the subsequent analysis pools data across the two matching rules, while still controlling for different matching rules in the parametric regression models.

high-value bidder dropped out. These censored bids are at least as high as this drop-out price, so the minimum bid consistent with these prices (displayed on Figure 3) presents only the lower bound of the intended bid by this low-value bidder.[6] The statistical tests below account for this censoring.

In all panels of these figures, the modal bid equals the bidder's value. Overbidding by the low-value bidder, however, is pronounced in Figure 3. About one-half of all bids exceed 700 (55 percent in the ascending-bid auction and 49 percent in the second-price sealed-bid auction). Conditional on overbidding, the figure suggests that more aggressive bids such as 750 and 790 are more common in the sealed-bid auction. This is reflected in the overall average bid submitted by the low-value bidder, which is 696 in the ascending-bid auction and 719 in the sealed-bid auction. Figure 4 suggests that underbidding is more common than overbidding for the high-value bidder in the sealed-bid auction.

Figure 5 summarizes the bid combinations for the complete information sealed bid auctions in the treatment in which pairs of bidders are randomly re-assigned each period. The modal bid pair is on the dominant strategy equilibrium (700, 800), but other pairs are common. Most of the pairs lie to the right of the line drawn on the surface of this diagram. This line indicates where the low-value bid equals the high-value bid. Therefore, the high-value bidder nearly always wins the auction even though many bids deviate from the dominant strategy equilibrium.

Figure 6 presents the time series path of bids for three example fixed pairs of bidders in this complete information sealed bid environment. Because of these fixed pairings, subjects could react directly to each other's bids in the previous periods.[7] Some pairs (not shown on this figure) often played the dominant strategy equilibrium, but many other pairs frequently changed their bids across rounds as illustrated by the three pairs in Figure 6. Typically they remain below the Bid1=Bid2 line that distinguishes the efficient and inefficient allocations. Pairs were also quite heterogeneous. For example, Pair 2 exhibited substantial counter-spiteful behavior (even leading to two cases where the low-value bidder won), whereas Pair 3 did not exhibit any underbidding by the high-value bidder.

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[6] The censoring problem is much greater for the high-value bidder, since in the ascending-bid auction this bidder wins in 278 of the relevant 309 auctions. Therefore, we do not report a bid distribution for the high-value bidder for this auction institution, nor do we use such bids in any of the statistical tests that follow.

[7] Subjects switched roles randomly between the 700 and 800 value, however.

Figures 7 and 8 summarize the bids for the incomplete information environment. Recall that values are drawn from $U[500,800]$. The figures display bids separately for the buyer with the highest and the lowest value draws, although subjects only observed their own value draw and therefore did not know their ranking. For reference the figures indicate a solid line where bid=value. Again, we do not include the ascending-bid auction bids for the highest value bidder, since this bidder nearly always won the auction and so his bid is typically not observed.

Careful inspection of the figures should remind the reader that bids were constrained to 10-yen intervals, while value draws could correspond to any integer yen amount. Therefore, by design the auctions will frequently feature bids that do not equal values. Overbidding and underbidding appear about equally common on the figures, although on average bids slightly exceed values (by less than one percent).

4.2 Hypothesis Testing: Complete Information Environment

This section reports tests of the hypotheses generated by the complete information model in Sections 2.1 and 2.2.

Hypothesis H1: *In the complete information environment, (a) low-value bidders overbid relative to their values, and (b) overbidding is more common for low-value bidders than for high-value bidders.*

Figure 3 shown above illustrates widespread overbidding for the low-value bidders. This indicates support for H1. To document how widespread this overbidding is across subjects, we determined how frequently individuals bid above their value when they had the low value draw in the complete information environment, across both auction institutions. Thirty-nine of the 84 subjects (46 percent) submitted bids greater than their values in *at least one-half* of these cases. In other words, nearly half of the subjects submitted bids that exceeded their value at least half of the time when they knew that they had the lower value draw. By contrast, only 8 out of 84 subjects (10 percent) submitted bids that were less than their values in at least half of these opportunities.

Table 1 reports results from several random-effects regression models (with subjects as the random effect) to formally test part (b) of Hypothesis H1. These models include a dummy variable to indicate when the bid is submitted by the lower value bidder, and they also control for any time trend (using 1/period) and for the fixed versus random matching of bidding pairs. The estimates only use the sealed bid auction data, since as already noted the ascending price bids for the high-value bidder are heavily censored because this bidder typically wins.

The regression shown in column 1 indicates that bids relative to values are not significantly different between the low-value and the high-value bidders. The difference (Bid – Value) averages 20 yen for the low-value bidder and 19 yen for the high-value bidder. By contrast, column 2 indicates that the likelihood of overbidding is much higher for low-value bidders. Low-value bidders overbid 49 percent of the time, whereas high-value bidders overbid only 20 percent of the time. This difference is highly significant and provides strong support for Hypothesis H1. The theoretical model's predictions are based on agents who have spiteful preferences. Therefore, columns 3 and 4 present estimates for the subset of subjects who bid above their value at least half the time when they had the low value draw. These 39 subjects represent roughly half the sample and their bids most clearly reveal spiteful preferences. The results indicate that these frequent over-bidders do not overbid arbitrarily, but instead they overbid more frequently when they have the lower value, consistent with Hypothesis H1.

Hypothesis H2: *In the complete information environment, (a) low-value bidders bid higher in the second-price sealed-bid auction than in the ascending-bid auction, and (b) overbids (especially large overbids) are more common in the second-price sealed-bid auction than in the ascending-bid auction.*

The figures and the summary statistics presented above provide some suggestive evidence in support of H2. For a systematic statistical test, however, we must account for the censoring of the bids in the ascending-bid auction. Recall that for this institution we do not observe the bid of the winning bidder—only the price at which the other bidder drops out. This censoring occurs for 30 of the 291 (10 percent) of the low-value bidders' bids.

We employ survival analysis to account for this censoring. This statistical methodology is common in fields such as medicine (where it gets its name) when the complement to survival—failure—is death. In economics it is used, for example, to study the duration of unemployment spells or strikes. In those cases, failure corresponds to finding a job or settling the strike. In the present application, "failure" occurs when the bidder drops out. The approach we use can account for differing censoring points, which occur when the other bidder drops out.

Figure 9 presents a comparison of the Kaplan-Meier nonparametric estimate of the survival function $S(x) = \text{Prob}(bid > x)$ for the two auction forms for the low-value bidders (e.g., see Cameron and Trivedi, 2005, Ch. 17). The median bid for the ascending-bid auction estimated using this method is 710, compared to 700 for the sealed bid auction. Overbidding (defined as any bid > 700) occurs with probability 0.58 in the ascending-bid auction, and with probability 0.49 in the sealed bid auction. The bid of 700, however, is the only place where the survivor function is higher for the ascending-bid auction. This is due to the higher mode of 700 in the sealed bid auction (cf Figure 3).

For all other bids < 800, the survivor function estimates imply that the sealed bid auction has a higher probability of observing bids exceeding all particular bid prices that are higher than 700. For example, if we define large overbid as a bid greater than or equal to 750, large overbidding occurs with probability 0.22 in the ascending-bid auction, and with probability 0.34 in the sealed bid auction. A log-rank test rejects the null hypothesis that these survivor functions are equal ($\chi^2_{1\text{ d.f}} = 5.56$; one-tailed p-value<0.01), and a parametric regression survival model (based on the exponential distribution and calculating standard errors to be robust to error clustering on individual subjects) also finds significant difference across treatments (t-statistic=2.40; one-tailed p-value<0.01). We therefore conclude that the data support Hypothesis H2, but only for the case of large overbids and not small overbids.

Since large overbids by low-value bidders are more common in the sealed-bid auction, a natural auxiliary hypothesis is that transaction prices are also higher in the sealed-bid auction:

Hypothesis H3: *In the complete information environment, (a) transaction prices are higher in the second-price sealed-bid auction than in the ascending-bid auction, and (b) prices above 700 (especially well above 700) are more common in the second-price sealed-bid auction than in the ascending-bid auction.*

Figure 10 indicates that the cumulative distributions of transaction prices for the two auction institutions are ordered consistent with Hypothesis H3, since the sealed-bid CDF is lower than the ascending-bid CDF for the critical range of prices between 710 and 790. Table 2 indicates, however, that when considering all prices the data fail to reject the hypothesis that prices are equal across institutions (model 1), or that high prices are equally likely in either auction institution (model 2). Many of the prices are in the range of 690 to 710, which occur when the low-value bidder adopts a value-revealing strategy. Therefore, in order to focus on bids that reflect considerable spitefulness, columns 3 and 4 report these same models after excluding the prices that are less than 711. Within this subset of data, which represents 38 percent of the price observations in columns 1 and 2, column 3 shows that transaction prices are significantly higher (by 11 yen) in the sealed-bid auction compared to the ascending-bid auction. Column 4 shows that the estimated likelihood that prices within this subsample exceed 740 increases from 25 percent in the ascending-bid auction to 54 percent in the sealed-bid auction. We therefore conclude that the data support the price differences indicated by Hypothesis H3, but only when excluding lower prices that arise from value-revealing bid strategies.

We conclude this subsection with a brief summary of auction efficiency. Recall that in equilibrium with spiteful preferences the low-value bidders bid less aggressively in the ascending-bid auction, relative to the sealed-bid auction, because the high-value bidders shade their planned bid (drop-out price) as the calling price rises above the low value. This is what leads to the lower predicted rate of large overbids in the ascending price auction. Note that it can also lead the low-value bidder to (inefficiently) win the auction less frequently in the ascending-bid auction than in the sealed-bid auction. Consistent with this pattern, in the complete information auctions the high-value bidder has the low bid in 11 percent of the ascending-bid auctions and 15 percent of the sealed-bid auctions. This difference is marginally significant based on a random effects probit model (t-statistic=1.41; one-tailed p-value<0.08). This suggests that the ascending-bid auction is marginally more successful in implementing efficient allocations.

4.3 Hypothesis Testing: Incomplete Information Environment

Section 2.3 established that even with spiteful preferences, in the incomplete information (unknown values) environment a unique symmetric equilibrium strategy exists where each bidder bids at her own value. Figures 7 and 8 indicate that significant dispersion of bids occurs both above and below value in the incomplete information data. Nevertheless, is not possible to reject the null hypothesis that a linear bid function fit on the incomplete information sealed bid data has an intercept of 0 and a slope of 1, consistent with the equilibrium model ($\chi^2_{2\text{ d.f}} = 1.02$; p-value=0.60). A more stringent test of the model, however, is that bids shift between the complete and incomplete information environments as predicted by spiteful preferences.

Hypothesis H4: *(a) Low-value bidders bid higher and (b) high-value bidders bid lower in the complete information environment compared to the incomplete information environment.*

This hypothesis is particularly demanding because in the incomplete information environment subjects do not know when they have the low or high value draw. They may have reasonably confident beliefs when they have very low value draws near 500 or very high value draws near 800, but not when they have intermediate values in the range between 600 and 700.

Since bids are typically not observed for the higher value bidder in the ascending-bid auction, to test this hypothesis we consider only the sealed bid auction where all bids are observed. In order to make the two environments comparable, we normalize all bids by subtracting value, and then regressing this difference on a dummy variable for the complete information environment, and the same control variables as in the regressions reported above. To be consistent with Hypothesis H4, the dummy variable for the complete information environment should be positive for low-value bidders (H4a) and negative for high-value bidders (H4b). The results, shown in Table 3, only indicate support for Hypothesis H4a, and only when restricting the analysis to the subsample of frequently overbidding subjects.

Another implication of the equilibrium result that bids should equal values in the incomplete information environment is that there should not be significant differences between bidding behavior for low and high value bidders.

Hypothesis H5: *In the incomplete information environment, overbidding is* **not** *more common for low-value bidders than for high-value bidders.*

This hypothesis is the counterpart of Hypothesis H1(b), where for the complete information environment the hypothesis was that overbidding *is* more common for low-value bidders than for high-value bidders. Recall that Table 1 presented models of bid deviations and overbidding that partially supported Hypothesis H1(b). The likelihood of overbidding is much higher for low-value bidders in the complete information environment, but the deviation between bid and value was not significantly different between low- and high-value bidders. Table 4 reports the identical models for the incomplete information environment, but this time the research hypothesis (H5) corresponds to the statistical null hypothesis that the dummy variable for the lower value is not significantly different from zero.[8] Consistent with Hypothesis H5, we find no evidence that bidding behavior is different for the low- and high-value bidders. Curiously, however, overbidding is significantly more common in the treatment with fixed pairings. We have no explanation for this result.

The final hypothesis is the incomplete information counterpart to Hypothesis H2. Recall that with complete information, overbids by the low-value bidder are predicted to be larger in the

[8] As in Table 1, we only employ the sealed bid auction data in these regressions because the high-value bidders' bids are rarely observed in the ascending price auction.

second-price compared to ascending-bid auction. By contrast, with incomplete information there should be no systematic difference between the bids across auction institutions.

Hypothesis H6: *In the incomplete information environment, (a) lower value bidders* **do not** *bid higher in the second-price sealed-bid auction than in the ascending-bid auction, and (b) overbids (especially large overbids) are* **not** *more common in the second-price sealed-bid auction than in the ascending-bid auction.*

We test Hypothesis H6 in exactly the same way that we tested Hypothesis H2. To account for the censoring of the bids in the ascending-bid auction, we again employ survival analysis. In the incomplete information environment, this censoring occurs for 30 of the 372 (8 percent) of the low-value bidders' bids. Figure 11 presents a comparison of the Kaplan-Meier nonparametric estimate of the survival function for the two auction forms for the low-value bidders. The median bid for the ascending-bid auction estimated using this method is one yen above value, compared to one yen below value for the sealed bid auction. Overbidding (defined as any bid > value) occurs with probability 0.54 in the ascending-bid auction, and with probability 0.40 in the sealed bid auction. There is virtually no evidence that large overbids are different for the two auction institutions, and the survivor functions are essentially identical for all bids that are 20 or more yen greater than value. Moreover, a log-rank test fails to reject the null hypothesis that these survivor functions are equal ($\chi^2_{1\text{ d.f}} = 2.03$; p-value=0.15). We therefore conclude that the data support Hypothesis H6: Overbidding by low value bidders in the incomplete information environment is not different in the sealed-bid and ascending-price auctions.

5. Conclusion

We have investigated bidding behavior in both complete and incomplete information environments for two-person second-price sealed-bid auctions and ascending-bid auctions for a single indivisible object with independent private values. Our intention-based bidding model features individuals who may be spiteful in the sense that they collect a positive psychological payoff when losing if they reduce the winners' payoff. Incorporating reciprocity as a basic motivation of individuals can result in distinctive bidding behavior in these two auction institutions under complete information.

When bidders have reciprocal preferences, a bidder who faces a spiteful bidder's over-bidding would retaliate by underbidding to increase the likelihood that the spiteful bidder wins and incurs a negative payoff. This possibility of counter-spite bidding causes spiteful bidders to refrain from bold overbidding. Our theoretical analysis concludes that the equilibrium bidding strategy differs from the Nash equilibrium strategy set generated without spite and counter-spite bidding, in the following three respects. First, the intention-based equilibrium strategy set is much smaller and does not contain any inefficient outcomes. Second, although a "bidding at one's value" strategy is no longer part of an intention-based equilibrium strategy profile, the equivalent outcome in which only the lower value bidder bids at her own value is one of the equilibrium outcomes. Third, the threat of counter-spite bidding is more important in ascending-bid auctions than second-price sealed-bid auctions, since a rising calling price reveals the spiteful intention of a losing, low-value bidder. This leads to a lower equilibrium spiteful over-bidding in ascending-bid auctions, which implies an even smaller equilibrium set with lower price upper bound.

Our experimental results are broadly consistent with the model's qualitative theoretical predictions. In the complete information setting, nearly half of the bids reflect spiteful overbidding. Bidders with lower private values are more prone to overbid in both auction formats, and their bidding behavior is more aggressive in the second price auctions than in the ascending-bid auctions. However, such systematic overbidding disappears when bidders' private values are random variables in the incomplete information setting, which is also consistent with the model.

Subjects' decision making seems to be different when they do or do not know each others' values. When they have complete information about all bidders' values, this allows them to evaluate their relative payoffs. A low-value bidder who bids 750, for example, knows that this bid will likely reduce the winning bidder's payoff by half relative to the payoff if all bids equal values. A bidder with the higher value can also perceive spiteful intentions of her opponent's bid in the complete information environment. It is this spiteful intention that induces a counter spiteful bid by the higher value bidder. This is the driving force behind our theoretical result that bidders make more timid overbids in the ascending-bid auction, because the rising calling price directly reveals the lower value bidder's spiteful intention. On the other hand, in the incomplete information setting bidders cannot assess their relative payoff position so their reciprocal motivation is not primed and they have much less incentive for spiteful overbidding.

Morgan et al. (2003) proposed a model with spiteful but not reciprocal bidders. Their investigation focuses on the incomplete information environment. In their model, bidders always overbid in both second price and ascending-bid auctions. Applying their model of preferences to the case of complete information yields a different equilibrium strategy set from that in our model. Their equilibrium set consists of only efficient outcomes as well, but does not contain the outcome which can also be generated by a value-revealing bidding strategy. Unlike ours, their model also predicts no difference in spiteful bidding behavior between the second price and the ascending-bid auction. Although their model is well-motivated empirically, they did not test their theoretical implications with field or laboratory data. Our experimental results do not support their predictions for either the complete or incomplete information environment. This can be interpreted as an additional evidence of negative reciprocity at work, but here in the context of an auction, consistent with negative reciprocity observed in the context of ultimatum and related games (e.g., see Charness and Rabin, 2002).

In zoology, it is well-known that reciprocal behavior is prevalent among variety of species, even including crabs (Hamilton, 1970). Animals that form herds can distinguish between self and others, and recognize relative positions between them. But according to Frith and Frith (1999), it is a distinctive characteristic among large apes such as chimpanzees and humans that individuals choose their actions or reactions by conjecturing intentions of others. This suggests that intention-based reciprocity may be embedded deep in human minds through evolution. Auction markets have been used for exchange since the birth of ancient civilizations, so it seems quite plausible that auction markets emerged as social institutions that function robustly with reciprocity and may even take advantage of humans' reciprocal motivations.

Table 1: Regression Models of Bid Deviations from Value and Overbidding: Complete Information Environment, Sealed Bid Auction

	All Bidders		Frequent Over-Bidders	
Model	1 (Random Effects GLS)	2 (Random Effects Probit)	3 (Random Effects GLS)	4 (Random Effects Probit)
Dependent Variable	Bid - Value	= 1 if Bid > Value	Bid - Value	= 1 if Bid > Value
Dummy Variable=1 if Lower Value	-0.38 (23.76)	1.42** (0.13)	2.22 (38.42)	1.70** (0.17)
Dummy Variable=1 for Fixed Pairings	22.51 (15.43)	0.29** (0.11)	39.75 (37.42)	0.18 (0.14)
1/period	70.65 (69.08)	-0.22 (0.19)	131.97 (153.40)	-0.36 (0.24)
Intercept	-18.36 (25.20)	-1.63** (0.23)	-24.54 (39.82)	-0.47* (0.23)
Observations	1116	1116	535	535
Number of Bidders	84	84	39	39
R^2 or Log-likelihood	0.01	-448.7	0.01	-258.3

Notes: Standard errors (in parentheses) are based on a subjects random effects model and for the GLS regressions in columns 1 and 3 are calculated to be robust to unmodeled correlation of choices within clusters defined by sessions.
* denotes significantly different from zero at the five-percent level, and ** denotes significantly different from zero at the one-percent level.

Table 2: Regression Models of Transaction Prices and Likelihood of High Prices: Complete Information Environment

	All Prices		Excluding Prices < 711	
Model	1 (Random Effects GLS)	2 (Random Effects Probit)	3 (Random Effects GLS)	4 (Random Effects Probit)
Dependent Variable	Price	= 1 if Price > 740	Price	= 1 if Price > 740
Dummy Variable=1 if Sealed-Bid Auction	-6.09 (5.46)	0.06 (0.12)	11.36** (3.07)	0.77** (0.16)
Dummy Variable=1 for Fixed Pairings	3.35 (4.60)	-0.01 (0.10)	-4.73 (2.91)	-0.17 (0.15)
1/period	1.87 (8.04)	-0.05 (0.17)	-3.98 (4.92)	-0.20 (0.25)
Intercept	713.10** (8.96)	-0.65** (0.20)	754.09** (3.59)	0.22 (0.18)
Observations	832	832	312	312
Number of Sessions	7	7	7	7
R^2 or Log-likelihood	0.00	-453.4	0.05	-178.7

Notes: Standard errors (in parentheses) are based on session random effects models.
* denotes significantly different from zero at the five-percent level, and ** denotes significantly different from zero at the one-percent level.

Table 3: Regression Models of Bid Deviations from Value to Compare the Complete and Incomplete Information Environments

	All Bidders		Frequent Over-Bidders	
Model	1 (Lower Value Bidders)	2 (Higher Value Bidders)	3 (Lower Value Bidders)	4 (Higher Value Bidders)
Dependent Variable	Bid - Value	Bid - Value	Bid - Value	Bid - Value
Dummy Variable=1 if Complete Info. Environment	0.94 (10.17)	11.36 (18.25)	16.89** (5.35)	34.84 (37.07)
Dummy Variable=1 for Fixed Pairings	2.33 (7.63)	24.87 (15.20)	1.10 (7.61)	59.47 (50.78)
1/period	-3.53 (7.37)	79.61 (75.12)	-7.58 (7.87)	193.48 (208.12)
Intercept	16.13 (8.95)	-34.02 (37.40)	32.49** (9.98)	-91.26 (101.83)
Observations	919	954	414	389
Number of Bidders	84	84	39	39
R^2	0.00	0.01	0.10	0.02

Notes: Standard errors (in parentheses) are based on a subjects random effects model and are calculated to be robust to unmodeled correlation of choices within clusters defined by sessions.
* denotes significantly different from zero at the five-percent level, and ** denotes significantly different from zero at the one-percent level.

Table 4: Regression Models of Bid Deviations from Value and Overbidding: Incomplete Information Environment, Sealed Bid Auction

	All Bidders		Frequent Over-Bidders	
Model	1 (Random Effects GLS)	2 (Random Effects Probit)	3 (Random Effects GLS)	4 (Random Effects Probit)
Dependent Variable	Bid - Value	= 1 if Bid > Value	Bid - Value	= 1 if Bid > Value
Dummy Variable=1 if Lower Value	3.43 (3.75)	0.003 (0.130)	5.07 (6.26)	0.011 (0.203)
Dummy Variable=1 for Fixed Pairings	3.79 (4.44)	0.93** (0.14)	3.93 (7.86)	1.26** (0.21)
1/period	-8.53** (3.19)	-0.82** (0.24)	-4.80 (6.69)	-0.55 (0.36)
Intercept	1.30 (3.37)	-0.88** (0.30)	12.74* (5.56)	0.02 (0.36)
Observations	757	757	268	268
Number of Bidders	48	48	17	17
R^2 or Log-likelihood	0.01	-316.8	0.01	-128.3

Notes: Standard errors (in parentheses) are based on a subjects random effects model and for the GLS regressions in columns 1 and 3 are calculated to be robust to unmodeled correlation of choices within clusters defined by sessions.
* denotes significantly different from zero at the five-percent level, and ** denotes significantly different from zero at the one-percent level.

Figure 1: Equilibrium Set of an Ascending-bid Auction without Spite or Counter-Spite

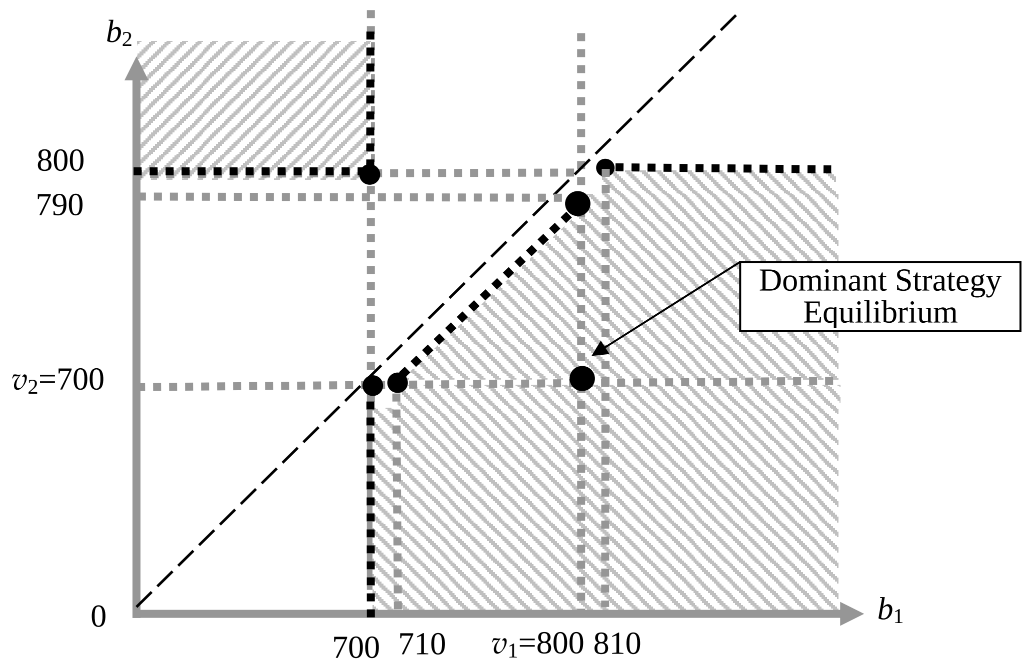


Figure 2: Example of Equilibrium Set in an Ascending-bid Auction with Spite and Counter-Spite Motivations

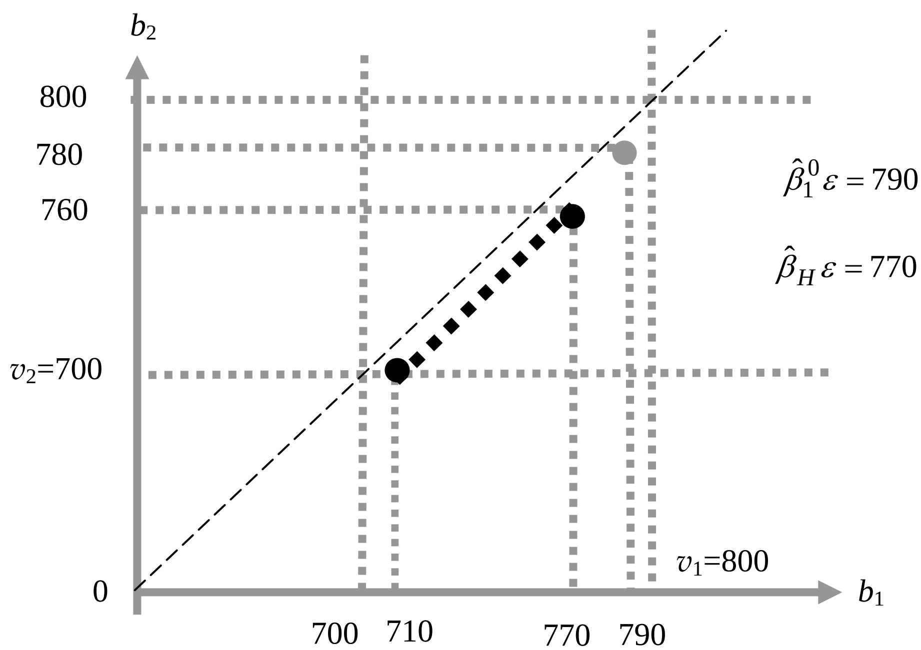


Figure 3a: Distribution of Ascending Price Auction Bids for Value=700

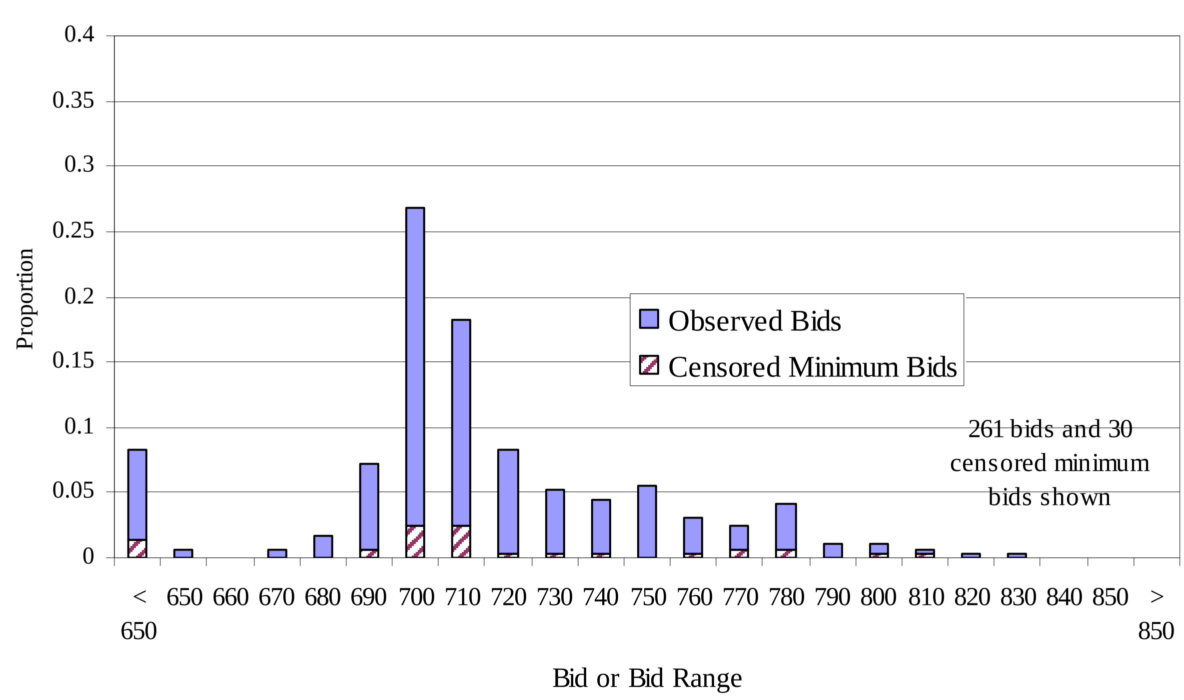
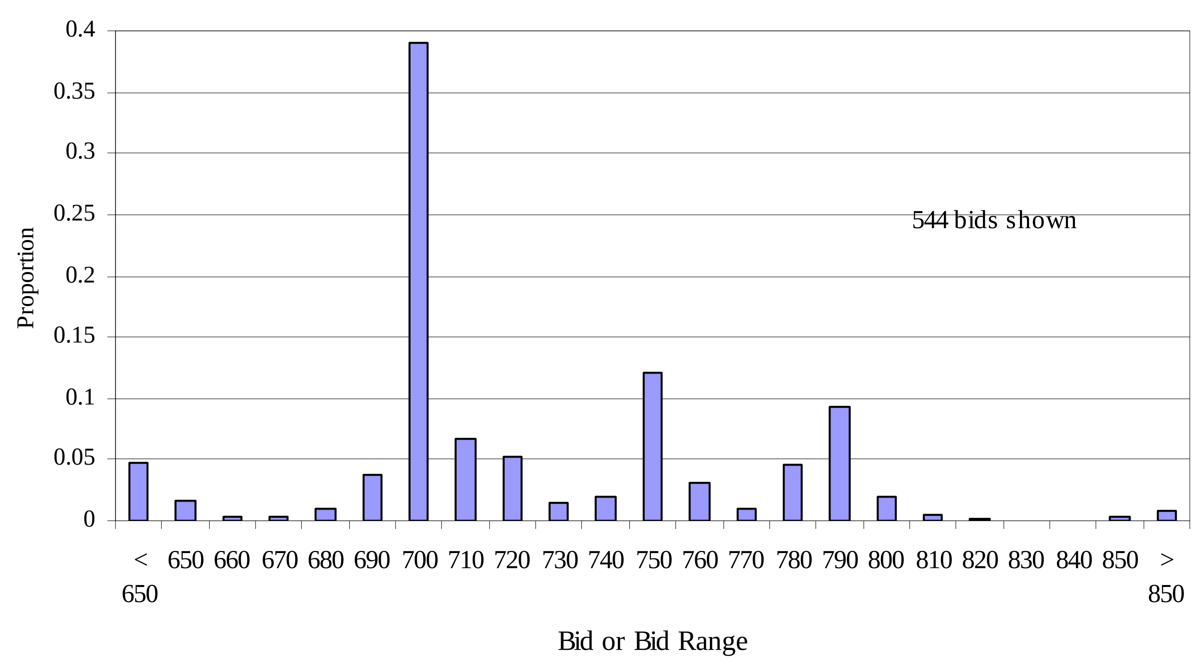


Figure 3b: Distribution of Second Price Sealed Auction Bids for Value=700



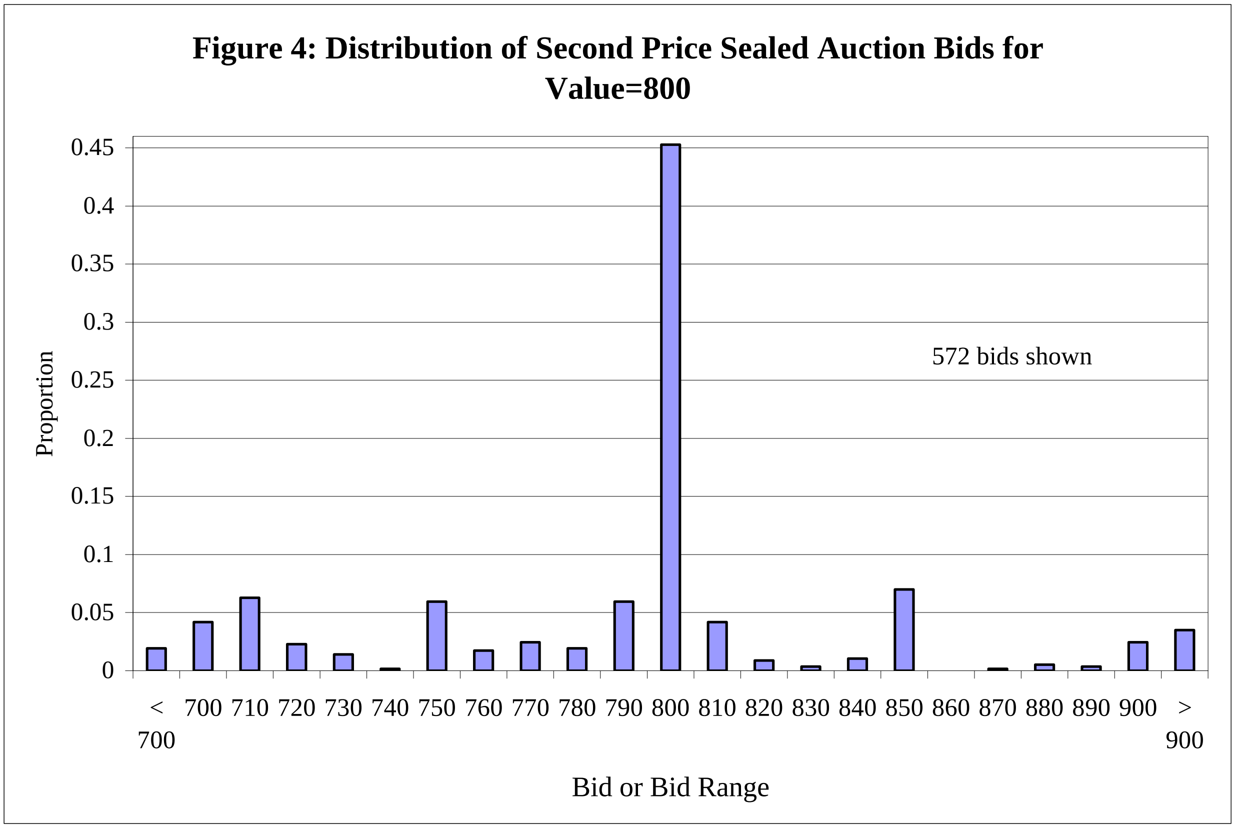


Figure 4: Distribution of Second Price Sealed Auction Bids for Value=800

Figure 5: All Bid Pairs for Random Groups Complete Information Second-Price Sealed Bid Auctions

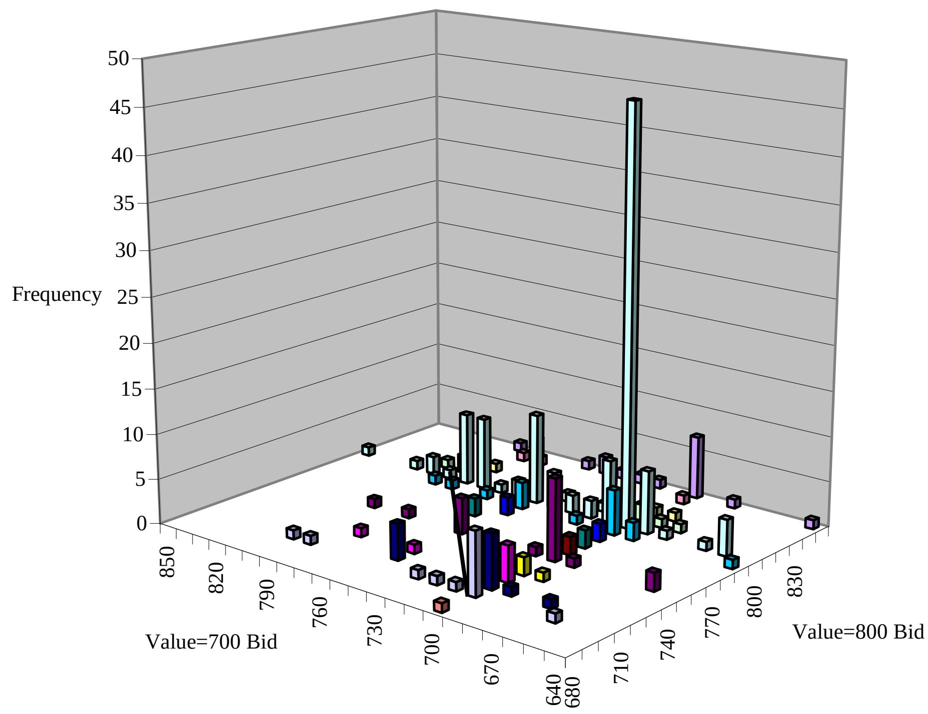


Figure 6: Example Fixed Pairs Sealed Bids in Complete Information Environment

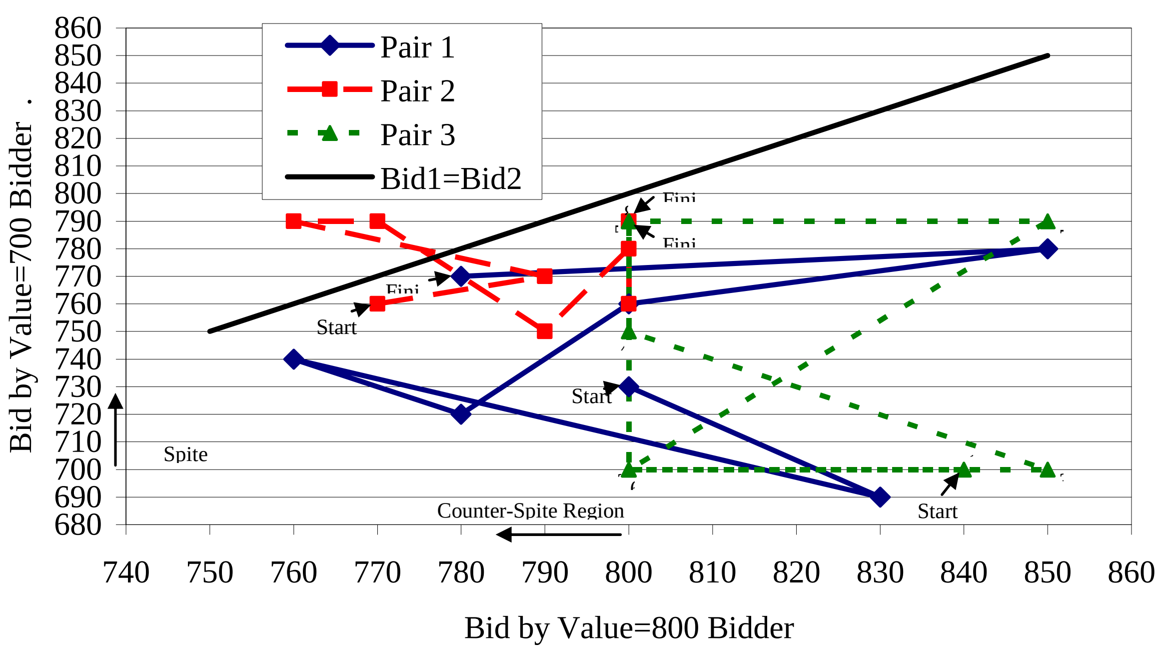


Figure 7a: Ascending Price Auction Bids for the Lower Value in Incomplete Information Environment

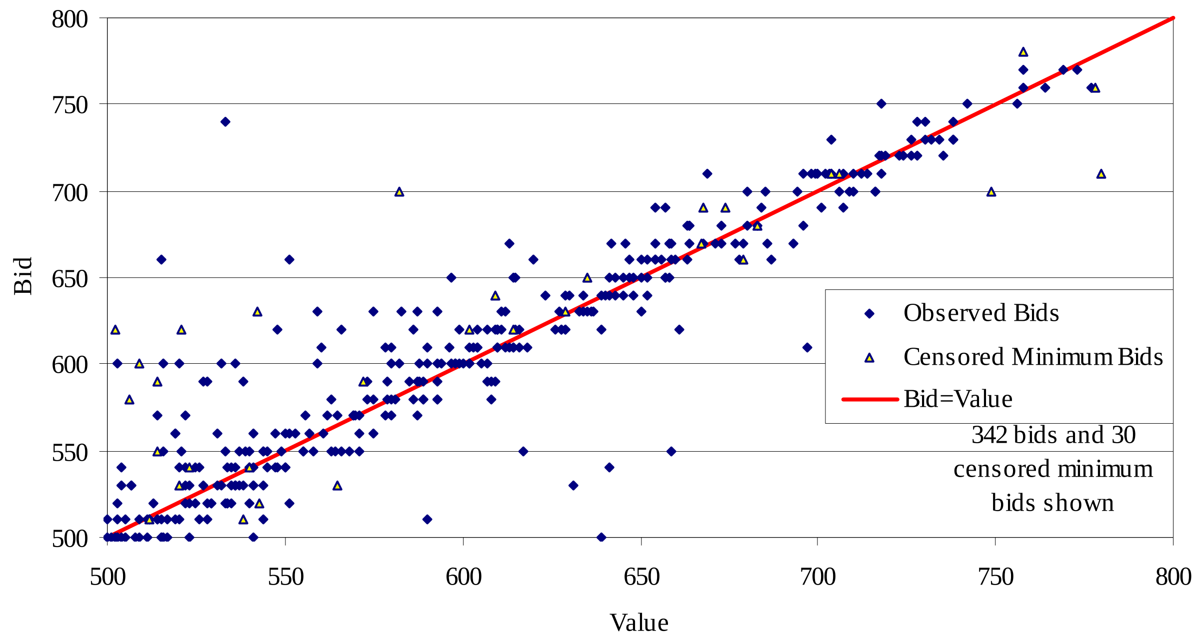


Figure 7b: Second Price Sealed Auction Bids for the Lower Value in Incomplete Information Environment

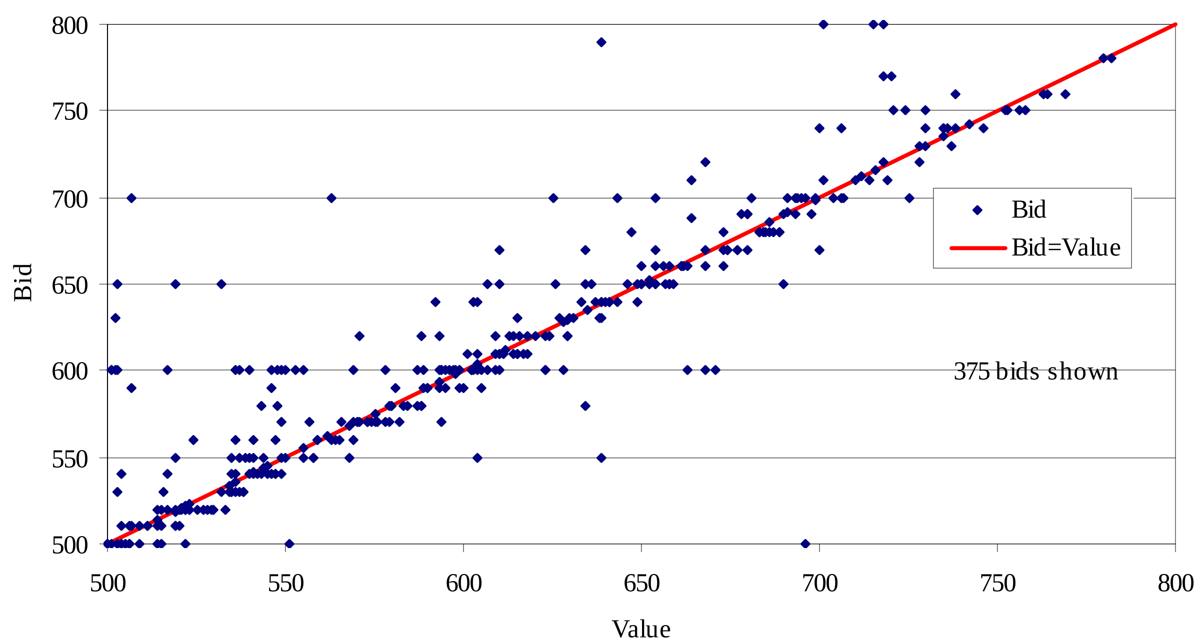


Figure 8: Second Price Sealed Auction Bids for Highest Value in Incomplete Information Environment

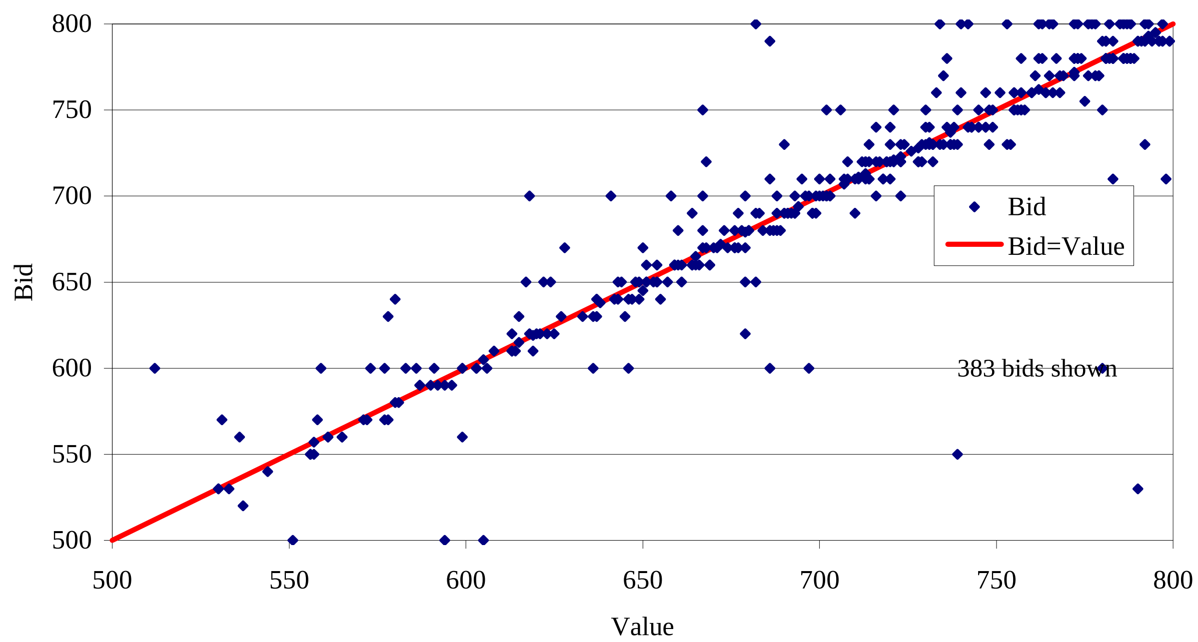


Figure 9: Comparison of Bid (Survivor) Functions for Complete Information with Value=700

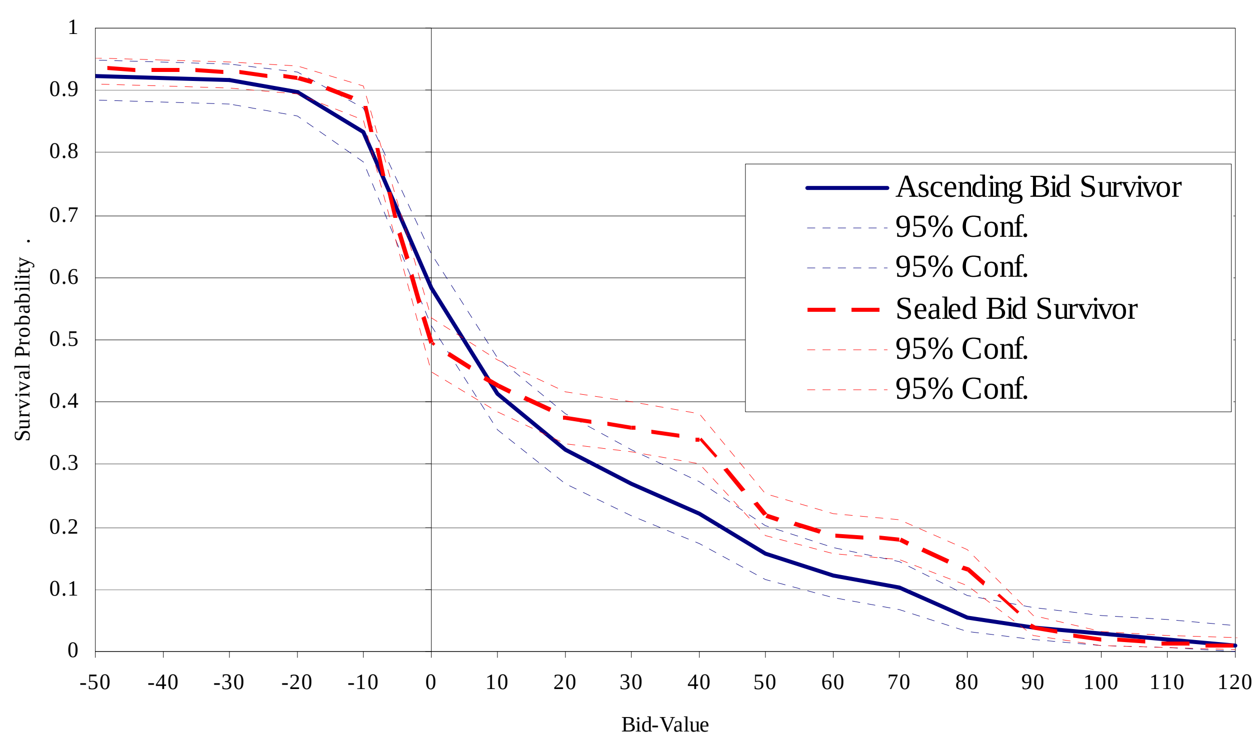


Figure 10: Cumulative Distribution Function of Transaction Prices for Complete Information Auctions

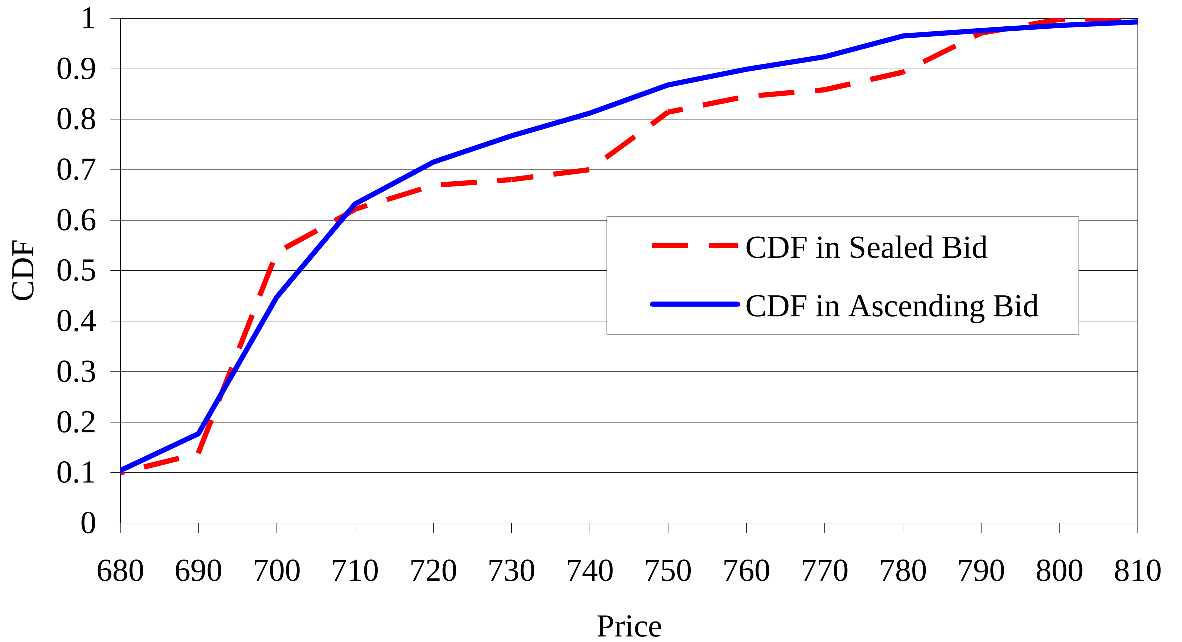
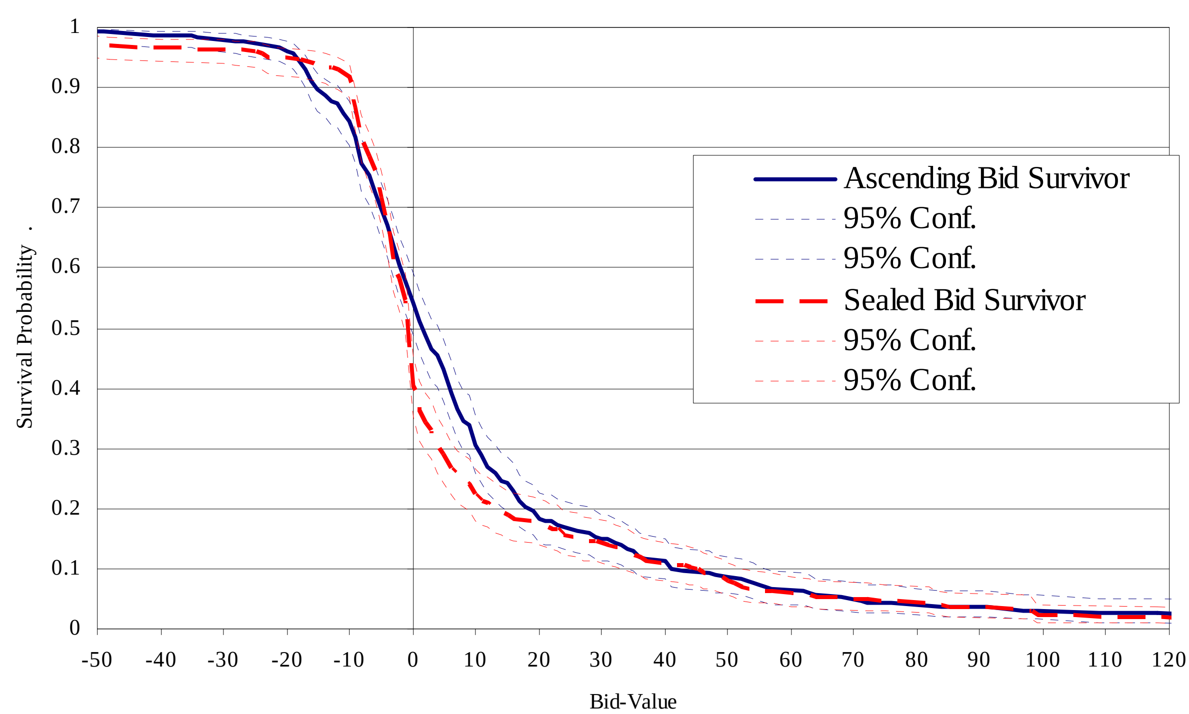


Figure 11: Comparison of Bid (Survivor) Functions for Incomplete Information Low-Value Bidder



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APPENDIX A

PROOF OF LEMMA 1a: From (2.19), it is immediate that one of the solutions to $\varphi_2(x,r)=0$ is $x=v_2$, irrespective of the value of *r*. Considering that the rule of the ascending-bid auction does not allow any bid below *r*, however, we notice that there is no solution to $\varphi_2(x,r)=0$ if $v_1 \le r$ because $\varphi_2(x,r)<0$ for all $x \ge r$. When $v_1 > r$ and $x \ge v_1$, we also have $\varphi_2(x,r)<0$ and again there is no solution. It comes down to consider the case $x<v_1$ and $v_1 - \varepsilon \ge r$. Rewrite equation (2.19) as

$$\varphi_2(x,r)=(v_2-x)\left[1-\left(\frac{\gamma_2\big(-(v_1-x)\big)}{\big(v_1-\max(r,v_2)+\varepsilon\big)^2}\right)\right].$$

The second parenthesis is positive for all $x \ge 0$. It follows that $\varphi_2(x,r)>0$ for all $x<v_2$, and $\varphi_2(x,r)<0$ for all $x \in (v_2,v_1)$. Thus, there is a unique solution to $\varphi_2(x,r)=0$, $x=v_2$.

PROOF OF LEMMA 1b: (i) (ii) (iii) Consider first the case $r>v_1$. By the rule of the ascending-bid auction, any bid *x* must satisfy $x \ge r$. Then, there is no solution to $\varphi_1(x,r)=0$, because $\varphi_1(x,r)<0$ for all $x \ge r$.

Second, consider the case $r \le v_1$. The relevant function in (2.18) is

$$\varphi_1(x,r)=[v_1-x]-(v_2-x)\left(\frac{\gamma_1\min(v_2-x,0)}{\big(v_1-\max\{v_2,r\}+\varepsilon\big)^2}\right). \tag{A.1}$$

When $x<v_2$ and $r<v_2$, (A.1) reduces to $\varphi_1(x,r)=v_1-x$, which is positive, and there is no solution for this bid range. Next, consider a bid level *x* such that $x \ge v_2$, while $r \le v_1$. The function $\varphi_1(x,r)$ in (2.18) becomes

$$\varphi_1(x,r)=[v_1-x]-\gamma_1\left(\frac{v_2-x}{v_1-\max\{r,v_2\}+\varepsilon}\right)^2, \tag{A.2}$$

Noting that $\varphi_1(v_2,r)>0$ and $\varphi_1(v_1,r)<0$, for any $r \le v_1$. Since the function $\varphi_1(\cdot,r)$ is continuous, by the mean value theorem, there exists a real number $z=\beta_1^r$ that satisfies $\varphi(z,r)=0$, located between v_2 and v_1. And such β_1^r is unique for a given $r \le v_1$, since

$\varphi_1(x,r)$ is a quadratic function of x with negative coefficient attached to x^2. In particular, at the terminal node r^*, $x = \beta_1^{r^*} = \beta_H$ is a solution to $\varphi_1(x,r^*) = 0$ with $r^* = x$, equivalently

$$0 = \varphi_1(x,x) = [v_1 - x] - \gamma_1\left(\frac{v_2 - x}{v_1 - \max\{v_2, x\} + \varepsilon}\right)\left(\frac{\min(v_2 - x, 0)}{v_1 - \max\{v_2, x\} + \varepsilon}\right). \tag{A.3}$$

It is immediate from (A.3) that $\beta_1^{r^*} = \beta_1^0$ for all $r^* \le v_2$, and $\beta_1^{r^*} = \beta_H = r^*$ for all $r^* \in (v_2, v_1]$.

(iv) (v) (vi) As $\varphi_1(\cdot,\cdot)$ is continuous in both arguments, applying the implicit function theorem to $\varphi_1(z,r) = 0$, we obtain

$$\frac{dz}{dr} = \frac{d\beta_1^r}{dr} = -\frac{\partial\varphi(z;r)/\partial r}{\partial\varphi(z;r)/\partial z} = -\frac{\dfrac{(v_2 - z)^2}{(v_1 - r + \varepsilon)}\left(-2\gamma_1\right)}{-(v_1 - r + \varepsilon)^2 + 2\gamma_1(v_2 - z)}. \tag{A.4}$$

Since $\beta_1^r \in (v_2, v_1)$, the sign of (A.4) is strictly negative, which means that β_1^r is strictly decreasing in *r*, and the lowest among β_1^r coincides with the one at the terminal node, where $\beta_1^r = r = \beta_H$, and we have $\beta_1^r < r$ for all $r \in (\beta_H, v_1]$, but it is of course not possible to bid at $\beta_1^r < r$ by the rules of the auction.

(vii) Applying the implicit function theorem to both (A.1) and (A.2), we examine a relationship between the solution of (A.1) and (A.2) respectively and the psychological payoff coefficient γ_1, by calculating

$$\frac{dz}{d\gamma_1} = \frac{d\beta_1^r}{d\gamma_1} = -\frac{\partial\varphi(z;r)/\partial\gamma_1}{\partial\varphi(z;r)/\partial z} = -\frac{-(v_2 - z)^2}{-(v_1 - r + \varepsilon) + 2\gamma_1(v_2 - z)},$$

and .

$$\frac{dz}{d\gamma_1} = \frac{d\beta_H}{d\gamma_1} = -\frac{\partial\varphi_1(x;x)/\partial\gamma_1}{\partial\varphi_1(x;x)/\partial x} = -\frac{-(v_2 - x)^2}{-(v_1 - x + \varepsilon)^3 + 2\gamma_1(v_2 - x)(v_2 - v_1 - \varepsilon)}.$$

Since any solution to (A.1) and (A.2) is located within the range (v_2, v_1), both of the above derivatives have a strictly negative sign.

PROOF OF LEMMA 1c: We need to examine whether the property obtained in lemma 1b remains valid when we restrict these solutions to be consistent with the minimum bid unit ε.

(i) For all $r \in \{0, \varepsilon, 2\varepsilon, \cdots, v_2\}$, it is obvious that $\hat{\beta}_1^r = \hat{\beta}_1^0$.

(iii) Lemma 1b (iv) implies that $\hat{\beta}_1^r \geq \hat{\beta}_1^{r'}$ for all $r, r' \in \{v_2 + \varepsilon, \cdots, v_1\}$ such that $r < r'$. And in particular, if $r = \beta_H = \hat{\beta}_H \varepsilon$, we have $\beta_1^r = \hat{\beta}_1^r \varepsilon = \hat{\beta}_H \varepsilon$. Otherwise, $\beta_H \in (\hat{\beta}_H \varepsilon, \hat{\beta}_H \varepsilon + \varepsilon)$.

(ii) (iv) If $r = \beta_H = \hat{\beta}_H \varepsilon$ and we have $\beta_1^r = \hat{\beta}_1^r \varepsilon = \hat{\beta}_H \varepsilon$, the assertion (ii) and (iv) follows from β_1^r being strictly decreasing in *r*. Suppose that $\beta_H \in (\hat{\beta}_H \varepsilon, \hat{\beta}_H \varepsilon + \varepsilon)$. Since β_1^r is strictly decreasing in *r* and $\beta_1^{\beta_H} = \beta_H, \beta_1^{\hat{\beta}_H \varepsilon + \varepsilon} < \hat{\beta}_H \varepsilon + \varepsilon$. This means that buyer 1 would never reach the decision point $r = \hat{\beta}_H \varepsilon + \varepsilon$. If $\beta_1^{\hat{\beta}_H \varepsilon} = \hat{\beta}_1^{\hat{\beta}_H \varepsilon} \varepsilon$, then $\beta_1^{\hat{\beta}_H \varepsilon} \varepsilon$ is the actual threshold bid. Consider the case $\beta_1^{\hat{\beta}_H \varepsilon} \neq \hat{\beta}_1^{\hat{\beta}_H \varepsilon} \varepsilon$. Since $\beta_H > \hat{\beta}_H \varepsilon$ and β_1^r is strictly decreasing in *r*, we have $\beta_1^{\beta_H} < \beta_1^{\hat{\beta}_H \varepsilon}$. Also $\beta_1^{\hat{\beta}_H \varepsilon} > \hat{\beta}_H \varepsilon$, since $\beta_1^{\beta_H} = \beta_H$ and $\beta_H > \hat{\beta}_H \varepsilon$. This leads to $\hat{\beta}_1^{\hat{\beta}_H \varepsilon} \geq \hat{\beta}_H \varepsilon$. Since $r = \hat{\beta}_H \varepsilon$ is the terminal node, for all $r \leq \hat{\beta}_H \varepsilon$, we have $\hat{\beta}_1^r \varepsilon \geq r$. Similarly, since we know from lemma 1b that $\beta_1^r < r$ when $r \geq (\hat{\beta}_H + 1)\varepsilon$, we have $\hat{\beta}_1^r < r$ for all $r \in \{(\hat{\beta}_H + 1)\varepsilon, \cdots, v_1\}$. But such $\hat{\beta}_1^r < r$ is not well defined under the rules of the ascending-bid auction.

(v) Suppose that $\beta_1^r = \hat{\beta}_1^r \varepsilon$ for some $\gamma_1 = \bar{\gamma}$. Then, since $\beta_1^r \in B$ is strictly decreasing in γ_1 from lemma 1b (vii), the corresponding threshold withdrawal bid when $\gamma_1 = \bar{\gamma} + e$ for small $e \in \Re_+$ should be equal or less than $(\hat{\beta}_1^r - 1)\varepsilon$. Next suppose that $\beta_1^r \in (\hat{\beta}_1^r \varepsilon, (\hat{\beta}_1^r + 1)\varepsilon)$ for some $\gamma_1 = \bar{\gamma}$. In this case, the threshold withdrawal bid when $\gamma_1 = \bar{\gamma}$ is again equal to $\hat{\beta}_1^r \varepsilon$.

The threshold withdrawal bid corresponding to $\gamma_1 = \bar{\gamma} + e$ is not necessarily less than $\hat{\beta}_1^r \varepsilon$, depending upon the magnitude of parameters, such as buyers' values, γ_1, and ε. Yet since $\beta_1^r \in B$ is strictly decreasing in γ_1, the threshold withdrawal bid is not increasing.

PROOF OF PROPOSITION 1: (i) (ii) In order to prove the proposition, we shall start by proving two lemmas. The first one concerns buyer 2's best response to anticipated buyer 1's bid under the consistent beliefs.

Lemma A1: Assume that buyer 2's beliefs satisfy the interim consistency requirement at any interim decision point $r \in B$.

(i) For all $r \in \{0, \varepsilon, \cdots, v_1 - \varepsilon, v_1\}$ and all $(b_{21}^r, b_{212}^r) \in B_1^r \times B_2^r$,

$$BR_2^r(b_{21}^r, b_{212}^r) = \begin{cases} \{b \in B_2^r \,|\, b > b_{21}^r\}, & \text{if } b_{21}^r \in \{r, r+\varepsilon, \cdots, v_2 - \varepsilon\} \\ \{b \in B_2^r \,|\, b \geq b_{21}^r\}, & \text{if } b_{21}^r = v_2 \\ \{b \in B_2^r \,|\, b = b_{21}^r - \varepsilon\}, & \text{if } b_{21}^r \in \{v_2 + \varepsilon, \cdots, \bar{b}\}. \end{cases}$$

(ii) For all $r \in \{v_1 + \varepsilon, \cdots, \bar{b}\}$ and $(b_{21}^r, b_{212}^r) \in B_1^r \times B_2^r$,

$$BR_2^r(b_{21}^r, b_{212}^r) = \{b \in B_2^r \,|\, b < b_{21}^r\} \text{ if } b_{21}^r \in \{r, \cdots, \bar{b}\}.$$

Proof of Lemma A1: It follows from lemma 1a that when $b_{21}^r \neq v_2$, it does not pay for buyer 2 to place a tie bid. Consider the case $r \in \{0, \varepsilon, \cdots, v_1 - \varepsilon, v_1\}$, where there is a relevant threshold bid. When $b_{21}^r \in \{r, r+\varepsilon, \cdots, v_2 - \varepsilon\}$, buyer 2 prefers to win. When $b_{21}^r \in \{v_2 + \varepsilon, \cdots, \bar{b}\}$, it is better off for buyer 2 to lose but place a maximum losing bid equal to $b_{21}^r - \varepsilon$, because her spiteful losing utility is increasing in her own bid. And when $b_{21}^r = v_2$, buyer 2 is indifferent between placing a tie bid and placing a winning high bid. Next consider the case $r \in \{v_1 + \varepsilon, \cdots, \bar{b}\}$. Then, it is obvious from (2.19) that buyer 2 always prefer to lose by placing any lower bid.

Similarly, the next lemma states buyer 1's best response.

Lemma A2: Assume that buyer 1's beliefs satisfy the interim consistency requirement.

(i) For all $r \in \{0, \varepsilon, \cdots, v_1 - \varepsilon, \hat{\beta}_H \varepsilon\}$ and all $(b_{21}^r, b_{212}^r) \in B_1^r \times B_2^r$,

if $\beta_1^r = \hat{\beta}_1^r \varepsilon$,

$$\text{then} \quad BR_1^r(b_{12}^r, b_{121}^r) = \begin{cases} \{b \in B_1^r \,|\, b \geq b_{12}^r + \varepsilon\}, & \text{if } b_{12}^r \in \{r, r+\varepsilon, \cdots, (\hat{\beta}_1^r - 1)\varepsilon\} \\ \{b \in B_1^r \,|\, b \geq b_{12}^r\}, & \text{if } b_{12}^r = \hat{\beta}_1^r \varepsilon \\ \{b \in B_1^r \,|\, b = b_{12}^r - \varepsilon\}, & \text{if } b_{21}^r \in \{(\hat{\beta}_1^r + 1)\varepsilon, \cdots, \bar{b}\}, \end{cases}$$

and if $\beta_1^r \neq \hat{\beta}_1^r \varepsilon$

$$\text{then} \quad BR_1^r(b_{12}^r, b_{121}^r) = \begin{cases} \{b \in B_1^r \,|\, b \geq b_{12}^r + \varepsilon\}, & \text{if } b_{12}^r \in \{r, r+\varepsilon, \cdots, \hat{\beta}_1^r \varepsilon\} \\ \{b \in B_1^r \,|\, b = b_{12}^r - \varepsilon\}, & \text{if } b_{12}^r \in \{(\hat{\beta}_1^r + 1)\varepsilon, \cdots, \bar{b}\}. \end{cases}$$

(ii) For all $r \in \{(\hat{\beta}_H + 1)\varepsilon, \cdots, v_1, \cdots, \bar{b}\}$ and all $(b_{21}^r, b_{212}^r) \in B_1^r \times B_2^r$,

$$BR_1^r(b_{12}^r, b_{121}^r) = \{b \in B_1^r \,|\, b < b_{12}^r\}.$$

Proof of Lemma A2: (i) There are two cases to examine, when $\beta_1^r = \hat{\beta}_1^r \varepsilon$ and when $\beta_1^r \neq \hat{\beta}_1^r \varepsilon$. Let us examine the former case first. Suppose that bidder 1 expects bidder 2 to bid at $b_{12}^r = \hat{\beta}_1^r \varepsilon$, which is exactly the bidder 1's threshold bid characterized by lemma 1. Noting that buyer 1's losing utility is increasing in her own bid, her best response is to place a tie bid or a higher winning bid. Next suppose that buyer 1 expects b_{12}^r to be less than or equal to $\hat{\beta}_1^r \varepsilon - \varepsilon$. Then, $\varphi_1(x, r) > 0$ for $x \leq \hat{\beta}_1^r \varepsilon - \varepsilon$ and buyer 1 strictly prefers to win. Her best response is to place a bid strictly higher than b_{12}^r. Suppose that buyer 1 expects b_{12}^r to be larger than $\hat{\beta}_1^r \varepsilon$.

Then $\varphi_1(x,r)<0$ for $x>\hat{\beta}_1^r\varepsilon$ and she strictly prefers to lose. Since her losing utility is increasing in her bid, her best response is to place a bid exactly at the level of $b_{12}^r-\varepsilon$. Next let us examine the case where $\beta_1^r \neq \hat{\beta}_1^r\varepsilon$. There is no bid level available for buyer 2 that makes buyer 1 indifferent between winning and losing. Thus, buyer 1 strictly prefers to win when she expects buyer 2 to bid at $\hat{\beta}(r)\varepsilon$ or less, and otherwise she strictly prefers to lose by placing a bid exactly at $b_{12}^r-\varepsilon$.

(ii) When the calling price *r* exceeds v_1, buyer 1's utility consists only of her own monetary payoff. Then we are back to the conventional auction model, where we already know that buyer 1's best response to b_{12}^r exceeding v_1 is to place any lower losing bid, including withdrawing right at *r*. On the other hand, the last sentence of Lemma 1 (iii) asserts that there is no meaningful threshold value for the range $r\in\{(\hat{\beta}_H+1)\varepsilon,\cdots,\overline{b}\}$, because $\hat{\beta}_H\varepsilon$ is the maximum threshold bid level and bidding less than the calling price *r* is not possible under the rule of ascending-bid auction. Therefore, even when the calling price is below v_1, buyer 1's best response is to place any losing bid including *r*.

The above two lemmas directly facilitate the proof of Proposition 1.

Proof of Proposition 1: To prove the sufficiency part, suppose that a withdrawal bid profile $(\hat{b}_1^r,\hat{b}_2^r)\in B_1^r\times B_2^r$ satisfies the conditions stated in proposition 1. From lemma A1 and A2, it is easy to check that $\hat{b}_1^r\in BR_1^r(b_{12}^r=\hat{b}_2^r, b_{121}^r=\hat{b}_1^r)$ and $\hat{b}_2^r\in BR_1^r(b_{21}^r=\hat{b}_1^r, b_{212}^r=\hat{b}_2^r)$ at the same time. When beliefs satisfy the interim consistency requirement, this implies $(s_1(\hat{b}_1^r), s_2(\hat{b}_2^r))\in E_{AB}^r$.

Next consider the necessity part. Since each component of a pair of interim equilibrium strategies $(s_1(b_1^r), s_2(b_2^r)) \in E_{AB}^r$ is best response to each other under the consistent interim beliefs, the pair of equilibrium strategies should satisfy all the properties stated in Lemma 1, A1, and A2. It is easy to check that if a strategy pair $(s_1(b_1^r), s_2(b_2^r))$ is an equilibrium, $b_2^r = b_1^r - \varepsilon$ holds. Once r passes bidder 1's threshold bid $\hat{\beta}_1^r \varepsilon$, there is no interim equilibrium.

We need to pay attention to defining the range that equilibrium withdrawal bid can take, because the boundary of the range varies depending upon whether the exact threshold value $\beta_1^r \in \overline{B}$ for a given r is consistent with the minimum bid unit ε or not. Consider the case, first, where $\beta_1^r = \hat{\beta}_1^r \varepsilon$ holds. From lemma 1, A1, and A2, it follows that when $\beta_H = \hat{\beta}_H \varepsilon$, a withdrawal bid profile generated by an equilibrium strategy (b_1^r, b_2^r) satisfies $b_2^r = b_1^r - \varepsilon$ and $v_2 + \varepsilon \le b_1^r \le \hat{\beta}_1^r \varepsilon$, for all $r \in B$ such that $r \le \beta_H - \varepsilon$, and $E_{AB}^r = \emptyset$ for all $r \in B$ such that $r \ge \beta_H$. When $\beta_H \ne \hat{\beta}_H \varepsilon$, an equilibrium strategy pair satisfies $b_2^r = b_1^r - \varepsilon$ and $v_2 + \varepsilon \le b_1^r \le \hat{\beta}_1^r \varepsilon$ for all $r \in B$ such that $r \le \hat{\beta}_H \varepsilon$, and $E_{AB}^r = \emptyset$ for all $r \in B$ such that $r \ge (\hat{\beta}_H + 1)\varepsilon$.

Next consider the case $\beta_1^r \ne \hat{\beta}_1^r \varepsilon$. When $b_2^r = \hat{\beta}_1^r \varepsilon$, buyer 1 still prefers to win since the actual threshold is yet higher than her opponent's bid, i.e., $\beta_1^r > \hat{\beta}_1^r \varepsilon$. Thus, a withdrawal bid profile generated by an equilibrium strategy profile should satisfy that as $b_2^r = b_1^r - \varepsilon$ and $v_2 + \varepsilon \le b_1^r \le [\hat{\beta}(r) + 1]\varepsilon$, for all $r \in B$ such that $r \le \beta_H - \varepsilon$ when $\beta_H = \hat{\beta}_H \varepsilon$ and for all $r \in B$ such that $r \le \hat{\beta}_H \varepsilon$ when $\beta_H \ne \hat{\beta}_H \varepsilon$. And $E_{AB}^r = \emptyset$ for all $r \in B$ such that $r \ge \beta_H$, when $\beta_H = \hat{\beta}_H \varepsilon$, and for all $r \in B$ such that $r \ge (\hat{\beta}_H + 1)\varepsilon$ when $\beta_H \ne \hat{\beta}_H \varepsilon$.

PROOF OF LEMMA 2: Lemma 1 tells that if $r, r' \in \{v_2, v_2+\varepsilon, \cdots, v_1\}$ we have $\hat{\beta}_1^r \geq \hat{\beta}_1^{r'}$, and $\hat{\beta}_1^r = \hat{\beta}_1^{r'}$ if $r, r' \in \{0, \varepsilon, \cdots, v_2 - \varepsilon\}$, for any and $r' > r$. From Proposition 1, it means that the upper bound of the equilibrium withdrawal bid b_1^r is non increasing. Noting that $\bar{r} < v_1$ by definition and that the lower bound of equilibrium b_1^r is $\max\{r, v_2\}$, which is strictly increasing after a calling price r passes v_2, it is immediate that for any $r' > r$, $E_{AB}^r \supset E_{AB}^{r'}$ if $r, r' \in \{v_2, v_2+\varepsilon, \cdots, \bar{r}\}$, and $E_{AB}^r = E_{AB}^{r'}$ if $r, r' \in \{0, \varepsilon, \cdots, v_2 - \varepsilon\}$. Thus, the assertion follows.

PROOF of PROPOSITION 2: (i) To prove the sufficiency part, first suppose that $(s_1(b_1^*), s_2(b_2^*)) \in E_{AB}^{\bar{r}}$. Since Lemma 2 implies that $E_{AB}^{\bar{r}} = \bigcap_{r \in \{0,\varepsilon,2\varepsilon,\cdots,\bar{r}\}} E_{AB}^r$, $(b_1^*, b_2^*) \in E_{AB}^r$ for all $r \in \{0, \varepsilon, \cdots, \bar{r}\}$. Thus, (b_1^*, b_2^*) satisfies the equilibrium conditions described in Definition 2, i.e., $b_i^* \in BR_i^r(b_{ij}^r, b_{iji}^r)$, and $b_{ij}^r = b_j^*, b_{iji}^r = b_i^*$ for each $r \in \{0, \varepsilon, \cdots, \bar{r}\}$ and $i, j \in \{1,2\}$ with $i \neq j$. Therefore, $(b_1^*, b_2^*) \in E_{AB}$. Next, suppose that $(s_1(b_1^*), s_2(b_2^*)) \in E_{\bar{r}}$. Proposition 1 and Lemma 2 implies that $E_{\bar{r}} = \bigcap_{k \in \{\varepsilon,\cdots,0\}} E_k$. Noting that $\bigcup_{k \in \{0,\cdots,r\}} E_{AB}^{k-\varepsilon} = E_{AB}^0$, $(s_1(b_1^*), s_2(b_2^*)) \in E_{AB}^0$ as well, hence the initial consistency requirement is met and maintained through, i.e., $b_{ij}^0 = b_j^*$ and $b_{iji}^0 = b_i^*$. Thus, (b_1^*, b_2^*) satisfies the equilibrium conditions described in Definition 2.

In order to prove the necessity part, consider the case of $\hat{\beta} = \hat{\beta}_1^0 = \hat{\beta}_1^{\hat{\beta}_H \varepsilon}$, first. This is the case where the interim equilibrium sets do not experience any shrink as the calling price climbs above v_2, i.e., $E_{AB}^r = E_{AB}^0$ for all $r \in \{\varepsilon, \cdots, v_2, \cdots, \bar{r}\}$. Therefore any $(s_1(b_1), s_2(b_2)) \notin E_{AB}^{\bar{r}} \cup E_{\bar{r}}$ can not be equilibrium. Next, consider the case $\hat{\beta} \neq \hat{\beta}_1^{\hat{\beta}_H \varepsilon}$. Then, by Lemma 1, we can find some $\tilde{r}, r' \in \{v_2, v_2+\varepsilon, \cdots, \bar{r}\}$ such that $\tilde{r} < r'$ and $\hat{\beta}_1^{\tilde{r}} > \hat{\beta}_1^{r'} \geq \hat{\beta}_1^{\bar{r}}$.

Suppose that a strategy profile $(s_1(\tilde{b}_1), s_2(\tilde{b}_2)) \notin E_{AB}^{\bar{r}} \cup E_{\bar{r}}$ but $(s_1(\tilde{b}_1), s_2(\tilde{b}_2)) \in E_{AB}^{r'} \cup E_{r'}$. Considering that the interim equilibrium set contracts from both upper and lower bound, the pair of beliefs satisfies $(\tilde{b}_{ij} = b_{ij}^{r'}, \tilde{b}_{iji} = b_{iji}^{r'}) \geq (r, r)$, for $i, j \in \{1,2\}, i \neq j$, hence buyers do not revise their beliefs at the new and the last decision point $r = \bar{r}$. Noting, however, that $E_{\bar{r}} = \left\{(b_1, b_2) \in B_1 \times B_2 \middle| b_i \in \bigcup_{k \in \{\varepsilon, \cdots, \bar{r}\}} BR_i^{k-\varepsilon}(b_{ij}^{\bar{r}}, b_{iji}^{\bar{r}}), \text{and } b_i \leq \bar{r}, i, j \in \{1,2\}, i \neq j\right\}$, and $E_{AB}^{\bar{r}} = \bigcap_{r \in \{0, \varepsilon, 2\varepsilon, \cdots, \bar{r}\}} E_{AB}^{r}$ by Lemma 2, $(\tilde{b}_1, \tilde{b}_2)$ does not fulfill the equilibrium conditions required in Definition 2, and consequently $(s_1(\tilde{b}_1), s_2(\tilde{b}_2)) \notin E_{AB}^{w}$, which is a contradiction.

(ii) The assertion immediately follows from Lemma 2 and Definition 3.

PROOF OF COROLLARY: (i) It is implied by Lemma 1. (ii) It is straightforward from Lemma 2 and Proposition 2.

PROOF OF PROPOSITION 3: Consider a bidding function $b_r(\cdot)$ that is strictly increasing and $b_r(0) = 0$. At every decision point *r*, each bidder chooses an optimal $x \in V$ which maximizes the expected payoff of (2.26). Since $EU^r(x, v)$ is continuous in *x*, take the first derivative with respect to *x* at given v and we obtain

$$\frac{dEU^r(x,v)}{dx} = (v - b_r(x))\, g_v(x) \Big/ \left(1 - G_v(b_r^{-1}(r))\right)$$

$$- \left(\frac{x - b_r(x) - \max\{v - x, 0\}}{D^r(v,x)}\right) \gamma \left(\frac{\rho^r(v,x)}{D^r(v,x)}\right) \cdot g_v(z) / \left(1 - G_v(b_r^{-1}(r))\right)$$

$$+ \int_x^{\bar{v}} \left(\frac{-b_r'(x)}{D^r(v,z)}\right) \gamma \left(\frac{\rho^r(v,z)}{D^r(v,z)}\right) \frac{g_v(z)}{1 - G_v\left(b_r^{-1}(r)\right)} dz\,. \tag{A.5}$$

(i) Let us first examine the decision problem at $r \in \overline{B}$. Each individual's optimal symmetric equilibrium bidding strategy must satisfy the first order condition, which equates the derivative (A.5) at $x = v$ with 0. Noting that $D^r(v, v) = 0$ for all r, the condition is given by

$$0 = \left.\frac{dEU^r(x, v)}{dx}\right|_{x=v}$$

$$\Leftrightarrow (v - b_r(v)) \cdot g_v(v) + \int_v^{\bar{v}} \left(\frac{-b_r'(x)}{D^r(v, z)} \right) \gamma_i \left(\frac{-\max\{z - b_r(z), 0\}}{D^r(v, z)} \right) g_v(z) dz = 0. \qquad \text{(A.6)}$$

It is immediate that a bidding function $z = b_r(z)$ satisfies the equation (A.6), which proves that a value revealing bidding strategy generates a symmetric interim equilibrium.

To prove the sufficiency part, consider first a bidder whose private value is some value $\hat{v} \in \overline{V}$. Suppose that a bidding function $b_r(\cdot)$ prescribes him to bid below his own value, $b_r(\hat{v}) < \hat{v}$. If this function constitutes a symmetric equilibrium, this must satisfy (A.6) at $x = \hat{v}$. The first term of the LHS of (A.6) is positive. Noting that $\gamma > 0$, $b_r'(\cdot) > 0$ and $-\max\{z - b_r(z), 0\}$ is either negative or zero for all $z \in \overline{V}$, no matter what is the shape of $b_r(\cdot)$, the second term of the LHS of (A.6) is either positive or zero. It follows that the bidding function that specifies $b_r(\hat{v}) < \hat{v}$ for some $\hat{v} \in \overline{V}$ can not generate a symmetric interim equilibrium at any *r*. Suppose next that a bidding function $b_r(\cdot)$ specifies the bidder to bid over his own value, $b_r(\hat{v}) > \hat{v}$. In order for this function to satisfy the condition (A.6) for this bidder, there must be some $\hat{z} \in \overline{V}$ such that $b_r(\hat{z}) < \hat{z}$ so that the second term of the LHS of (A.6) has positive sign. If this function generates a symmetric interim equilibrium, the condition (A.6) must hold for $v = \hat{z}$ at the same time. However, we have already observed that any bidding function that prescribes to bid below value for at most one value level can not be a symmetric equilibrium strategy. Consequently a bidding function that maps any point in its domain to a different point can not generate a symmetric interim equilibrium at any *r*.

(ii) The above argument in (i) for each interim equilibrium directly applies to the optimal decision problem in the second price auction if we set $r = 0$. The first order condition for the optimal bidding decision in an ascending-bid auction is given by

$$0 = \left. \frac{dEU^r(x,v)}{dx} \right|_{\substack{x=v \\ b(x)=r}} .$$

This condition is equivalent to (A.6), hence we obtain the same conclusion described in (i) to the case of ascending-bid auction as well.

Appendix B: Experiment Instructions (Translation)

Thank you for participating in our experiment. This is a study on auctions. The Ministry of Science and Education has provided funds for our research. The instructions are simple. You are being paid 1,000 yen in cash as start-up money. All you have to do is to make a bid in each of the auction situations according to the rules described in this instruction. In each round of auctions, depending on the bid you make and the resolution of the uncertainty, you may receive or pay a specified amount of money as a result of transaction.

Your acts will be recorded and kept only in terms of purely anonymous data for the academic research on microeconomics.

Furthermore, and most importantly, this is not a project to see if you can make a "right" decision, or if you can come up with a "correct" answer.

INSTRUCTIONS

<1> WHAT ARE WE BIDDING FOR?

We will ask you to bid, not for a commodity as in a real auction market, but for a monetary prize. In other words, you are going to compete for the right to earn a monetary prize. In an exchange for the prize, a winner has to pay according to the rules of the auction.

At the beginning of each round of auction game, we will assign each of you a number. This represents a prize value to you. Once you win the auction, then you will receive the prize worth that value number, and you must pay the amount specified by the corresponding auction rules to obtain the prize. Your payoff is the difference between the prize and the amount of payment. If you do not win, you will not receive any prize and pay nothing, that is, your payoff is zero.

$$\boxed{\text{Your Payoff}} = \begin{cases} \boxed{\text{Your Assigned Prize Value}} - \boxed{\text{Payment,}} & \text{if you win,} \\ \boxed{\text{None,}} & \text{if you do not win.} \end{cases}$$

You know your prize value for sure once it is assigned to you. But you may or may not know the value assigned to the other participants in this experiment, depending on the experimental design.

<2> THE AUCTION RULES

We will run two kinds of auctions. One is a sealed bid auction, called the second-price auction, and the other is an open bid auction, called the ascending-bid auction. The experiment starts with a session of sealed bid auctions, containing eight rounds of auctions each with fresh value assignment, and then proceeds to a session of ascending-bid auctions also containing eight rounds.

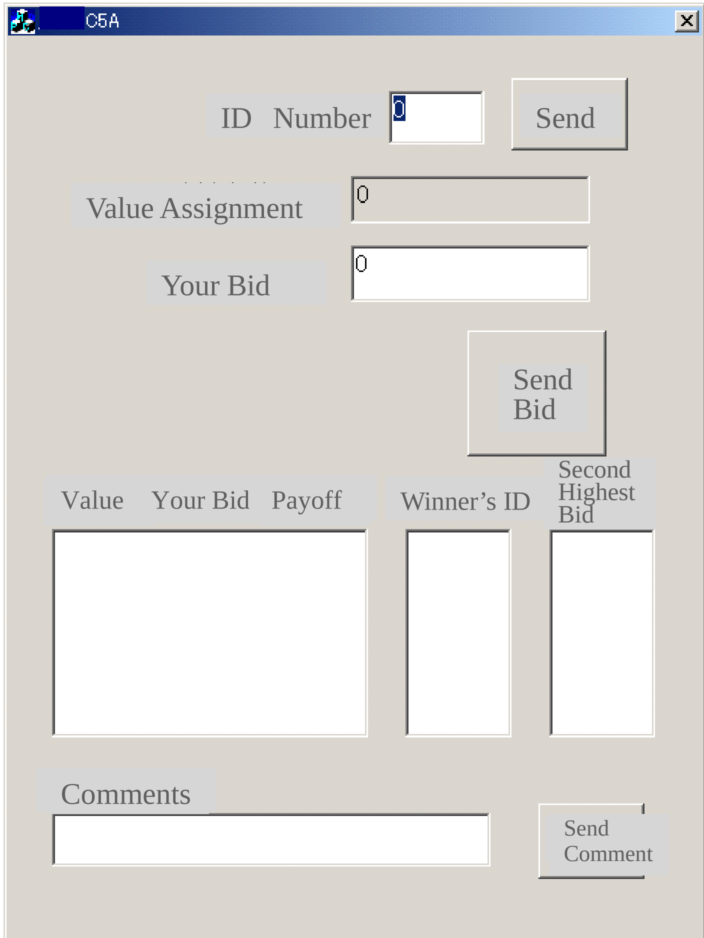
(1) THE SECOND-PRICED SEALED BID AUCTION

Each of you is paired with another anonymous participant. Every auction round starts with the value assignment. You and the other participant receive a number each as your prize value, which we call "assigned value." After receiving the assigned value, both of you are asked to make a bid. That is, you have to specify a number to submit to the experimenters. We collect those submitted bids, and identify a bidder who submitted the higher bid between the two of you, as a winner. The winner receives the right to obtain her assigned value. **In exchange, the winner has to pay the amount equal to the lower bid in the pair, that is, the bid amount the non-winner submitted. Please note that the payment required by the winner is not her own bid.**

You know that you are paired with someone, but you do not know who is paired with you. There are two treatments as to pairing. In one treatment, you are paired with a different person, randomly determined, in every round. We call this treatment, "Part 1." In the other treatment, you are paired with a person randomly in a first round, and continue to bid against this same anonymous person in the rest of the rounds. We call this treatment "Part 2." In Part 1, we start with the second price auction for eight rounds, and then proceed to "Part 2" for eight rounds. After that, we conduct the ascending-bid auction experiments under the treatment of Part 1 and then Part 2.

Upon being instructed to do so, the first thing you have to do is to double click the icon indicated by "Sealed Bid" on your windows screen. Then you will see the dialogue window shown below, popping up in your screen.

Make sure that you see the header "Sealed Bid" on the left of the dialogue window. Your ID number will be shown in a box in the first line of the dialogue window. **Do not let the other participant know your ID number.** This is very important to maintain the academic quality of our experiments. And **please do not close this window by yourself**. Once every necessary step is complete, the window will automatically close.



Next, the experimenter will send you a private "assigned value," which will appear in the box of the dialogue window labeled "Value Assignment." In the event you win, this is the prize you are entitled to earn by making the appropriate payment.

Only after the assigned values are distributed, the face of the "send bid" button turns black and you can then type the amount you decide to bid in the box labeled "Your Bid." **The minimum bid unit is 10 yen.** If you are sure about that amount, then click the "send bid" button to transmit that information to the experimenter. Your bid won't reach the experimenter unless you click the "send bid" button. After all bids are transmitted, the system identifies the bidder whose submitted bid is the higher in the pair as a winner, and lets her know that she is the winner.

You will find the winner's ID number shown in the box labeled "Winner's ID" in the middle part of the dialogue window. The further right box under the header "the second highest bid" will show the amount that the winner has to pay, which is the lower bid in the pair. If you are the winner, the further left box under the header "payoff" will show the number that is your payoff obtained by subtracting the payment from your assigned value. On the other hand, if you are not the winner, the number in the box under the “payoff” is zero, since you do not get the prize and do not have to make any payment.

For example, consider the case where your assigned value is 400 yen. Suppose that you bid 300 yen, and the other participant paired with you bids 350 yen. In this case, you are not the winner since your bid is not the highest, and your payoff is zero.

Next, suppose that the other participant paired with you bids 200 yen instead. Your bid, 300 yen, is the highest bid and you become the winner. Then you win the prize of 400 yen but must pay 200 yen, which comes down to the payoff of 200 yen.

If you win,

$$\boxed{\text{Your Payoff}} = \boxed{\text{Assigned Value (¥400)}} - \boxed{\text{The Second Highest Bid (¥200)}} = \boxed{\text{¥200}}$$

If you do not win,

$$\boxed{\text{Your Payoff}} = \boxed{\text{¥0}}$$

If you and the other participant bid the same amount, then a winner will be randomly selected. In this case, you will be a winner with probability of 50%.

In the previous example, suppose that two of you bid 300 yen. Then, you can obtain the prize of 400 yen and make the payment of 300 yen with probability of 50%, and obtain zero payoff otherwise.

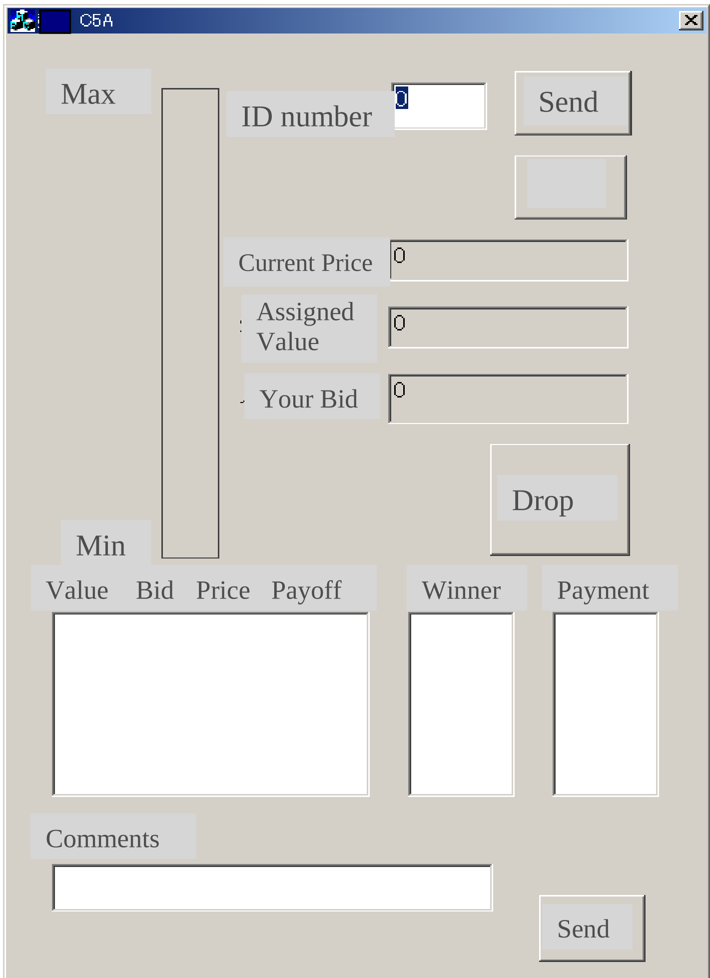
$$\boxed{\text{Your Payoff}} = \begin{cases} \boxed{\text{Assigned Value (¥400)}} - \boxed{\text{The Second Highest Bid (¥300)}}, & \text{with 50\% chance,} \\ \boxed{\text{¥0,}} & \text{with 50\% chance.} \end{cases}$$

In the very bottom of the dialogue window, you find the “comments” box. Please type why and how you have come to a bid decision, when we, the experimenters, ask you to do so. Having finished typing your comments, click the “send comments” button. Your comments won’t be sent to the experimenters unless you click the “send comments” button.

(2) THE ASCENDING-BID AUCTION

In the ascending-bid auction, we, the experimenters, raise a price gradually from a very low level. At the beginning of the ascending-bid auction, all of you are "active" in the sense that you are bidding at that price level. Unless you indicate that you wish to withdraw from bidding, you are considered being active and willing to pay that amount of indicated price in the event that you become the winner at this very moment. As long as two of you are active, we continue to raise the price. At the moment when one bidder withdraws from bidding, then the remaining bidder becomes the winner awarded with the prize, and she has to pay the last price level at which two bidders were active.

After you double click the icon named "Ascending Bid" on your screen, you will see the following dialogue window.



Please make sure that your ID number is shown in the box located at the top of the dialogue window. **DO NOT let the other participants know of your ID.**

Similar to the experiment of the second-price sealed bid auction, you will see your assigned value pop up in the corresponding box. If you become a winner, this is the amount you will get as a prize.

Underneath that box, there is a box indicating the “current price”. Once the auction starts, the number shown in that box rises gradually from 0 yen. As the number increases, the price thermometer on the left of the dialogue window grows higher. The price increases by 10 yen.

Let us consider the case where the number in the “current price” box is 100. Suppose that you are willing to pay 100 yen if you win at this moment, but you would not want to pay more than 100 yen. In this case, **click the “Drop” button to indicate that you wish to withdraw from bidding when the number in the “current price” box gets up to 110**. Your withdrawal bid level will be recorded as 100. Unless you click the “Drop” button, your wish to withdraw would not be transmitted to the experimenter.

As long as the “current price” increases, the other participant paired with you is active. When one of the pair drops, the process of the ascending-bid auction stops. If the price stops increasing before you click the “Drop” button, this means that you win. Then, you will obtain the prize worth your assigned value, and your payment is 100 yen, which is one unit (=10 yen) lower than the price level at which the process stops. The payment amount will be indicated in the “payment” box. The ID number of the winner will be shown in the “winner” box. Your payoff will be indicated under the large box in the middle under the header of “payoff.” If you win, your payoff is your prize minus the payment. If you do not win, your payoff is zero. Such information will be listed in that box in each round. Each new result enters at the top of the list.

Let us review the above details by some examples. Suppose that your assigned value is 400 yen. Suppose that the number in the “current price” box increases and stops at 350 before you click the “drop” button. This means that you win, and your payment is one step earlier than the 350 level, which is 340 yen. Your payoff is 400 minus 340, which is 60 yen.

$$\boxed{\text{Your Payoff}} = \boxed{\text{Your Assigned Value (400)}} - \boxed{\text{Price before the Stop (340)}} = \boxed{60}$$

Suppose that you click the “drop” button at 360. It means that you do not win, and your payoff is zero.

When two of you simultaneously click the “drop” button, then a winner will be randomly selected, and the winner pays the price at the moment of withdrawal minus 10.

At the bottom of the dialogue window, there is the “comments” box. Describe why and how you figure out the amount of price at which you choose to withdraw, when instructed to do so. To transmit your comments to the experimenter, click the “send” button.

<3> THE EXPERIMENT PROCEDURE

After you sit down in front of the computer terminal, each of you will be given ID number, which you continue to use during the full course of experiment. Do not show that number to any other participant. It is very important for the academic quality of this experiment that you keep your ID number completely private.

(1) PAIRING

Having mentioned elsewhere, you are paired with another person among those participants in this room. You will never be told with whom you are paired. In Part 1, the person you paired with will be determined randomly every round of the auction, while in Part 2, your paired person will be determined randomly in the first round, and maintained the same anonymous person through out all rounds.

(2)VALUE ASSIGNMENT

There are two ways in which your value is assigned in a round. In one case, which we call "treatment VA1," your value is selected randomly from a predetermined set of values, which consists of 700 and 800, every round. Your computer screen informs you of your own value only, but if you receive 700, it automatically implies that your paired participant receives 800, and vice versa.

In the other case, which we call "treatment VA2," your value is a purely random variable from a predetermined range of value of [500, 800] with uniform distribution. In every round, a fresh value is drawn independently. Again, you are informed of your own value only, and never be informed of the realized value drawn for your paired person. But the same random procedure is applied to both of you and your paired person, independently.

In order to get familiar with the auction rules, the first four rounds are for your practice. The outcomes from the subsequent rounds are recorded for real prize and payment.

Please make sure that you fully understand the rules and procedure. You will be given a short quiz after all the instruction is completed.

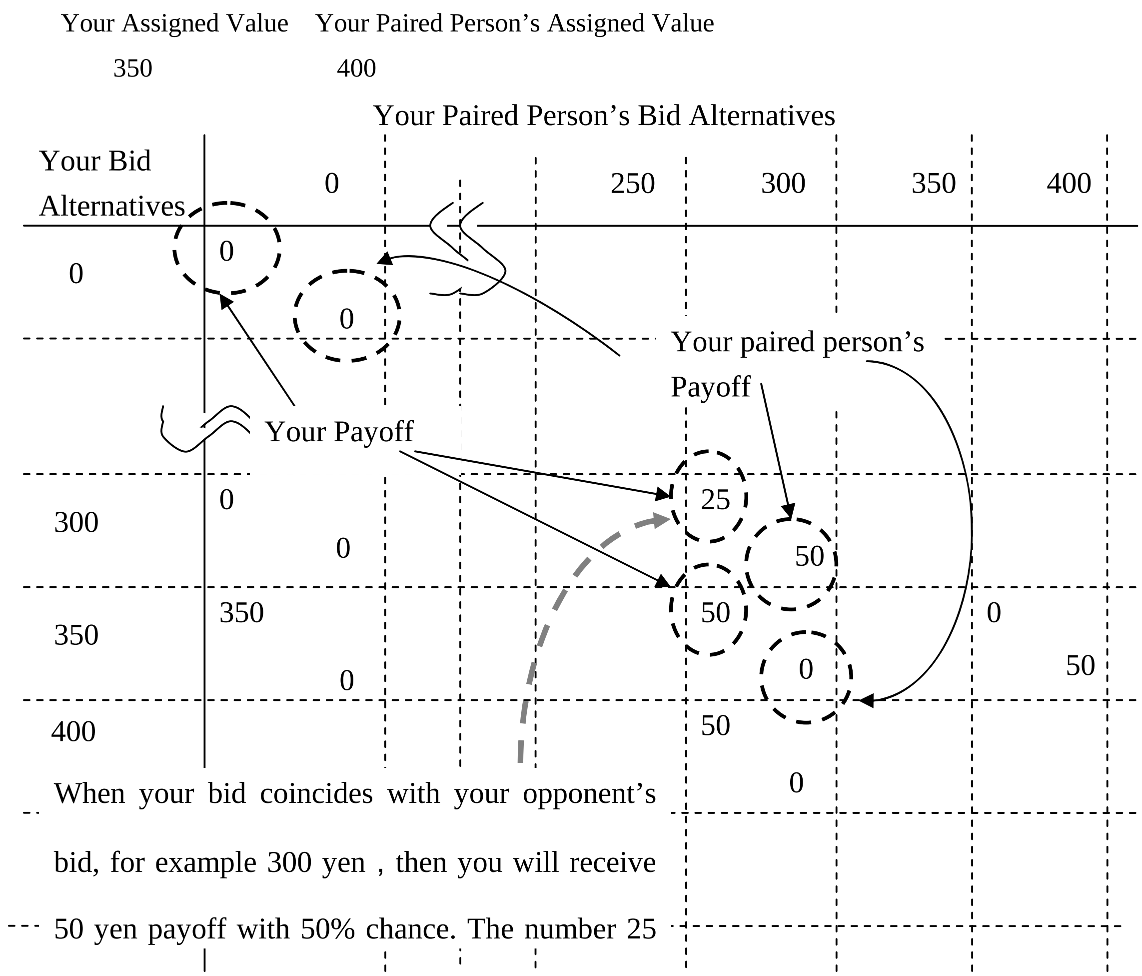
(3) HOW TO USE PAYOFF TABLE

Your payoff is a joint product of your own bid choice and a bid chosen by the other participant paired with you. The other participant's payoff is also a joint product. Though there are numerous combinations of your bid and the other participant's bid, in the treatment VA1, we provide you a payoff table that looks like the figure shown below. The table lists your payoffs and the other participant's payoffs under the various but limited number of bid combinations, because of the space limit.

The figure below shows the example of the payoff table with your assigned value being 350 yen and the other participant's being 400 yen. The further left column lists the possible bids you can choose, and the first row lists the bids available for your paired person. Though those possible bids are listed with 50 yen increments, you are free to bid in 10 yen increments in the auction.

There are two numbers shown in each cell. The number in the upper left of the cell is your payoff and the number in the lower right of the cell is the other participant's payoff.

For example, suppose that your bid is 350 yen and the other participant bids at 300 yen. Then, your bid is higher and you are the winner. You will be awarded 350 yen prize, your assigned value, and you have to pay 300 yen, the other participant's bid level, so that your net payoff is 50. The other participant who loses receives zero net payoff.



Let us consider another case. Suppose that you bid 350 yen and the other participant bids 400 yen.

Since her bid is higher, she is the winner and gets 400 yen prize and pays 350 yen equal to your bid. Her net payoff is 50, subtracting 350 from 400, which is shown in the lower right of the cell in the row of your 350 bid and the column of the other participant's bid 400.

Another important case is the event of tie. Suppose that you bid 300 yen and the other participant bids also 300 yen. Then the winner will be randomly selected. That is, you become the winner with 50% chance, receiving your value prize of 350 yen and paying 300 yen. Your net payoff is 50 yen in when you win, and your expected payoff is 25 yen, which is shown in the upper left of the corresponding cell. Since the other participant's assigned value is 400 yen, her expected payoff is 50=0.5*(400-300), which is shown in the lower right of the corresponding cell.

<4> THE SUMMARY OF THE COURSE OF THE EXPERIMENTS

There are two parts regarding paring; Part 1 is the treatment where you are paired with a randomly selected person among other participants every round. Part 2 is the treatment where you are paired with a randomly selected person in the first round and maintain the same person in the rest of the rounds.

There are two treatments regarding value assignment. In treatment VA1, you are assigned either 700 or 800 yen randomly, and in treatment VA2, you are assigned with a random number drawn from the range between 500 and 800 yen according to the uniform distribution. Each draw is independent.

There are two types of auctions; one is the second-price sealed bid auction and the other is the ascending-bid auction, and the sealed bid auction precedes the ascending bid auction.

As a total, there are 2x2x2 variations in our experimental treatments. We will run on average 8 rounds for each treatment. At the beginning of the first sealed-bid auction experiment, there are four rounds set as the practice session. All payoffs generated during the practice session will not be counted. After completing the practice rounds, then we move on to the first set of 8 rounds of auction and start recording the realized payoffs.

Your payoffs will be all recorded. At the end of the experimental session, we will pay you the cumulative amount in cash, on the spot.

<5> NOW, WE ARE READY TO START.

Please read this instruction carefully. It is very important that you understand these instructions. Should you have any questions, please feel free to ask us.

We would like you to leave this room after the experiment with as much money as possible.

Once we start explaining the instructions, you are not allowed to talk to any other participants. You can only talk to us, the experimenters, if necessary. You are not allowed to look at the other participants' PC screens. This no talking and no peeking code is very important for the validity of our experiments. Not conforming to this code would jeopardize the quality of our experiments.